Annexe A: New/Revised Course Content in OBTL+ Format

Course Overview

The sections shown on this interface are based on the templates UG OBTL+ or PG OBTL+
If you are revising/duplicating an existing course and do not see the pre-filled contents you expect in the subsequent sections e.g. Course Aims, Intended Learning Outcomes etc. please refer to Data Transformation Status for more information.

Expected Implementation in Academic Year	AY2024-2025
Semester/Trimester/Others (specify approx. Start/End date)	Semester 2
Course Author * Faculty proposing/revising the course	Leonard Huang
Course Author Email	leonard.huang@ntu.edu.sg
Course Title	Linear Algebra II
Course Code	MH1201
Academic Units	4
Contact Hours	51
Research Experience Components	Not Applicable

Course Requisites (if applicable)

Pre-requisites	MH1200 Linear Algebra I
Co-requisites	
Pre-requisite to	
Mutually exclusive to	MH2800, MH2802, CY1602
Replacement course to	
Remarks (if any)	

Course Aims

This first-year course is the second of two courses on linear algebra. It is a core module for MATH students. The course aims to develop your understanding of fundamental topics in linear algebra, with particular emphasis on abstract vector spaces and linear transformations.

Course's Intended Learning Outcomes (ILOs)

Upon the successful completion of this course, you (student) would be able to:

ILO 1	Identifying when a set of objects forms a vector space or vector subspace.
ILO 2	Identifying the span and determining the linear independence of a set of vectors.
ILO 3	Deriving a basis and computing the dimension of a vector space.
ILO 4	Describing and determining the null space and range of a linear transformation.
ILO 5	Applying the Rank-Nullity Theorem to determine properties of a linear transformation.
ILO 6	Solving for the eigenvalues, eigenvectors, and eigenspaces of a linear transformation.
ILO 7	Determining whether a given square matrix is diagonalizable.
ILO 8	Identifying when a set of objects forms an inner product space.
ILO 9	Applying Gram-Schmidt Orthogonalization to obtain an orthonormal basis.

Course Content

Abstract Vector Spaces: Real and complex vector spaces: definition and examples; subspaces.

Bases and Dimension: Linear combinations and span; linear independence; bases and coordinate vectors; dimension; finite-dimensional vector spaces; construction of bases.

Linear Transformations: Linear transformations and examples; null space and range; the Rank-Nullity Theorem; injectivity and surjectivity of linear transformations; bijections and isomorphisms.

Matrix Representations of Linear Transformations Between Finite-Dimensional Vector Spaces: Examples of linear transformations and their matrix representations; change-of-bases; applications.

Eigenvalues and Eigenvectors: Eigenvalues and eigenvectors of matrices and linear transformations; characteristic polynomials; eigenspaces; diagonalization; applications.

Inner Product Spaces: Inner products and norms; inner product spaces; the Cauchy-Schwarz Inequality; orthogonality; orthonormal sets, Gram-Schmidt Orthogonalization; orthogonal projections; orthogonal complements; applications.

Reading and References (if applicable)

Sheldon Axler, *Linear Algebra Done Right*, 4th Edition, Springer, 2024.

ISBN: 978-3-031-41025-3.

Book website: https://linear.axler.net/.

The e-book can be legally downloaded for free.

Gilbert Strang, *Introduction to Linear Algebra*, 6th Edition, Wellesley-Cambridge Press, 2023.

ISBN: 978-17331466-7-8.

NOTE: These two readings comprise the foundational readings for the course, and more up-to-date relevant readings will be provided when they are available.

Planned Schedule

Week or Session	Topics or Themes	ILO	Readings	Delivery Mode	Activities
1	Vector Spaces - Definition of a Vector Space	1	Chapter 1A, 1B	In-person	Lectures
2	Vector Spaces - Linear Subspaces	1	Chapter 1C	In-person	Lectures, tutorial, and in-class quiz
3	Finite-Dimensional Vector Spaces - Span and Linear Independence	2, 3	Chapter 2A	In-person	Lectures, tutorial, and in-class quiz
4	Finite-Dimensional Vector Spaces - Bases and Dimension	2, 3	Chapter 2B, 2C	In-person	Lectures, tutorial, and in-class quiz
5	Linear Transformations - the Vector Space of Linear Transformations	4, 5	Chapter 3A	In-person	Lectures, tutorial, and in-class quiz
6	Linear Transformations - Null Space and Injectivity, Range and Surjectivity, the Rank-Nullity Theorem	4, 5	Chapter 3B	In-person	Lectures, tutorial, and in-class quiz
7	Linear Transformations - Matrices, Representing a Linear Transformation by a Matrix, Matrix Multiplication	4, 5	Chapter 3C	In-person	Lectures, tutorial, in-class quiz, and Midterm Exam

Week or Session	Topics or Themes	ILO	Readings	Delivery Mode	Activities
8	Linear Transformations - Invertible Linear Transformations, Isomorphic Vector Spaces, Matrix Multiplication and Composition of Linear Transformations	4, 5	Chapter 3D	In-person	Lectures, tutorial, and in-class quiz
9	Eigenvalues and Eigenvectors - Solving for Eigenvectors and Eigenvalues, Characteristic Polynomials, Diagonalization	6, 7	Chapter 5B, 5C	In-person	Lectures, tutorial, and in-class quiz
10	Eigenvalues and Eigenvectors - Characteristic and Minimal Polynomials, the Jordan Canonical Form	6, 7	Chapter 8B, 8C, 8D	In-person	Lectures, tutorial, and in-class quiz
11	Inner Product Spaces - Inner Products, Norms, the Cauchy-Schwarz Inequality	8, 9	Chapter 6A	In-person	Lectures, tutorial, and in-class quiz
12	Inner Product Spaces - Orthogonality, Orthonormal Bases, Gram-Schmidt Orthogonalization	8, 9	Chapter 6B	In-person	Lectures, tutorial, and in-class quiz

Week or Session	Topics or Themes	ILO	Readings	Delivery Mode	Activities
13	Review for the Final Exam	1-9		In-person	Lectures, tutorial, and in-class quiz

Learning and Teaching Approach

Approach	How does this approach support you in achieving the learning outcomes?
Lectures (39 hours)	Derivation and demonstration: - Explains the motivation behind linear-algebraic concepts. - Presents systematic ways to solve problems related to linear-algebraic concepts being developed. - Derives methods to systematically find bases, eigenvalues, eigenvectors, and orthonormal bases. Problem-solving sessions: - Develops competence in solving a variety of problems related to linear algebra.
Tutorials (12 hours)	Derivation and demonstration: - Explains the motivation behind certain linear-algebraic concepts. - Presents systematic ways to solve problems related to linear-algebraic concepts being developed. - Derives methods to systematically find bases, eigenvalues, eigenvectors, and orthonormal bases. In-class quizzes: - Develops competence in solving a variety of problems related to linear algebra. Peer instruction: - Hones communication and presentation skills while deepening understanding. You will have the opportunity to work with your peers and to present your ideas to the class.

Assessment Structure

Assessment Components (includes both continuous and summative assessment)

No.	Component	ILO	Related PLO or Accreditation	Weightage	Team/Individual	Rubrics	Level of Understanding
1	Continuous Assessment (CA): Test/Quiz(Midterm Exam)	1, 2, 3, 4, 5		25	Individual	Analytic	Multistructural
2	Summative Assessment (EXAM): Final exam(Final Exam)	1, 2, 3, 4, 5, 6, 7, 8, 9		60	Individual	Analytic	Multistructural
3	Continuous Assessment (CA): Class Participation(In-Class Quizzes)	1, 2, 3, 4, 5, 6, 7, 8, 9		15	Team	Holistic	Multistructural

Description of Assessment Components (if applicable)

The weekly in-class quizzes are designed to assess the following:
- Your understanding of course concepts, demonstrated by your ability to recall and apply standard linear-algebraic facts in solving linear-algebraic problems.
- Your skill in formulating proofs of basic linear-algebraic propositions that are not necessarily covered in the lectures and tutorials but fall well within the scope of the course.
- Your ability to collaborate with your peers in the social activity of mathematical problem solving.

The Midterm Exam is designed to assess the following:
- Your understanding of course concepts, demonstrated by your ability to recall and apply standard linear-algebraic facts in solving linear-algebraic problems.
- Your skill in formulating proofs of basic linear-algebraic propositions that are not necessarily covered in the lectures and tutorials but fall well within the scope of the course.

The Final Exam is designed to assess the following:
- Your overall understanding of course concepts, demonstrated by your ability to recall and apply standard linear-algebraic facts in solving linear-algebraic problems.
- Your skill in formulating proofs of basic linear-algebraic propositions that are not necessarily covered in the lectures and tutorials but fall well within the scope of the course.

Formative Feedback

Formative feedback will be provided in the following manner:
1. During each in-class quiz, you will work with your peers on specific problems. After each problem, you will communicate your solution to your tutor, who will then provide immediate feedback.
2. Feedback is given after the Midterm Exam on common mistakes made on the exam.

NTU Graduate Attributes/Competency Mapping

This course intends to develop the following graduate attributes and competencies (maximum 5 most relevant)

Attributes/Competency	Level
Collaboration	Basic
Creative Thinking	Basic
Problem Solving	Intermediate
Sense Making	Intermediate
Transdisciplinarity	Intermediate

Course Policy

Policy (Academic Integrity)

Good academic work depends on honesty and ethical behaviour. The quality of your work as a student relies on adhering to the principles of academic integrity and to the NTU Honour Code, a set of values shared by the whole university community. Truth, Trust and Justice are at the core of NTU's shared values. As a student, it is important that you recognize your responsibilities in understanding and applying the principles of academic integrity in all the work you do at NTU. Not knowing what is involved in maintaining academic integrity does not excuse academic dishonesty. You need to actively equip yourself with strategies to avoid all forms of academic dishonesty, including plagiarism, academic fraud, collusion and cheating. If you are uncertain of the definitions of any of these terms, you should go to the academic integrity website for more information. On the use of technological tools (such as Generative AI tools), different courses / assignments have different intended learning outcomes. Students should refer to the specific assignment instructions on their use and requirements and/or consult your instructors on how you can use these tools to help your learning. Consult your instructor(s) if you need any clarification about the requirements of academic integrity in the course.

Policy (General)

You are expected to diligently attend all lectures, whether in-person or online, and attempt in-class activities. While you are not expected to complete all assigned tutorial problems prior to a tutorial session, please be aware of the topics that will be covered and participate wholeheartedly in discussions with your group-mates. A general observation: Students who struggle together do well together.

Policy (Absenteeism)

Past data demonstrates a strong correlation between in-class participation and the final course grade. You are therefore expected to attend all lectures and tutorials.

This course has a strict no-make-up policy for exams. If you miss the Midterm Exam due to a health condition, then please
(1) seek approval for short LoA (leave of absence) from your home school, and
(2) submit to both the course instructor and your home school a scanned copy of your original medical certificate (MC) issued in Singapore by a medical practitioner registered with the Singapore Medical Association.

Policy (Others, if applicable)

Diversity and Inclusion

Integrating a diverse set of experiences is important for a more comprehensive understanding of science.

It is our goal to create an inclusive and collaborative learning environment that supports a diversity of perspectives and learning experiences, and that honors your identities; including ethnicity, gender, socioeconomic status, sexual orientation, religion, or ability.

To help accomplish this:
- If you are neuroatypical or neurodiverse, have dyslexia or ADHD (for example), or have a social anxiety disorder or social phobia;
- If you feel like your performance in the class is being impacted by your experiences outside of class;

- If something was said in class (by anyone, including the instructor) that made you feel uncomfortable; then please speak to your teaching team, our school pastoral officer, or a peer or senior (either in-person or via email) about how we can help facilitate your learning experience.

As a participant in course discussions, you should also strive to honor the diversity of your classmates. You can do this by: using preferred pronouns and names; being respectful of others' opinions and actively making sure all voices are being heard; and refraining from the use of derogatory or demeaning speech or actions.

All members of the class are expected to adhere to the NTU anti-harassment policy. If you witness something that goes against this or have any other concerns, then please speak to your instructors or a faculty member.

Appendix 1: Assessment Rubrics

Rubric for Tutorials: In-Class Quizzes (15%)
Point-based marking.

Rubric for Lectures: Midterm Exam (25%)
Point-based marking.

Rubric for Examination: Final Exam (60%)
Point-based marking.