Annexe A: New/Revised Course Content in OBTL+ Format

Course Overview

The sections shown on this interface are based on the templates UG OBTL+ or PG OBTL+
If you are revising/duplicating an existing course and do not see the pre-filled contents you expect in the subsequent sections e.g. Course Aims, Intended Learning Outcomes etc. please refer to Data Transformation Status for more information.

Expected Implementation in Academic Year	AY2024-2025
Semester/Trimester/Others (specify approx. Start/End date)	Semester 1
Course Author * Faculty proposing/revising the course	Yue Mu
Course Author Email	mu.yue@ntu.edu.sg
Course Title	Probability & Statistics
Course Code	MH2814
Academic Units	3
Contact Hours	38
Research Experience Components	Not Applicable

Course Requisites (if applicable)

Pre-requisites	MH1810 OR MT1001
Co-requisites	
Pre-requisite to	
Mutually exclusive to	MT2001, CV2001, CV2018, HE1005, MH1820, MH2500
Replacement course to	
Remarks (if any)	

Course Aims

Uncertainties are unavoidable in the design and planning of engineering system. Therefore, engineering analysis should include probability and statistics to evaluate the significance of uncertainty on system performance and design. This course provides the basics of probability and statistical concepts in terms that are more easily understood by engineering students. We present probability and statistical concepts through problems that are meaningful to engineering science. This course should motivate the recognition of the significant roles of the relevance mathematical concepts in engineering.

Course's Intended Learning Outcomes (ILOs)

Upon the successful completion of this course, you (student) would be able to:

ILO 1	Use the techniques of permutations and combination to calculate the probability of the occurrence of an event
ILO 2	Apply Bayes' rule to compute conditional probability and identify independent events
ILO 3	State the definition of a random variable (RV), tabulate its probability mass function or density function, and be able to compute its expectation and variance
ILO 4	Recognize a joint probability distribution and compute its marginal and conditional distributions
ILO 5	Calculate the probability, mean and variance of a RV that has either binomial, geometric or Poisson distribution, and apply the Poisson approximation to binomial distribution
ILO 6	Evaluate the probability of a normal RV, its mean and variance, and apply the normal approximation to binomial distribution
ILO 7	Distinguish the difference between population and sample, parameter and statistic
ILO 8	Apply the Central Limit Theorem (CLT) to the distribution of sample mean
ILO 9	Construct confidence interval for sample mean, and determine the sample size with appropriate margin of error
ILO 10	State the null and alternative hypotheses of a statistical test, determine the type I and type II errors, and compute its power and p-value
ILO 11	Apply a simple linear model to fit a dataset with 2 variables, calculate the values and construct confidence interval for the model coefficients, and perform prediction

Course Content

Topic 1: Basic Probability Theory: • Sample space, Events, Counting Sample Points, Permutations & Combinations. • Probability, Equally Likely Outcomes, Not Equally Likely outcomes, Useful Rules • Conditional Probability, Independent Events, Partition, Bayes' Rule

Topic 2: Random Variables & Probability Distribution: • Random Variables (RV), Discrete Random variables, Probability Mass Distribution, Cumulative Probability Distribution • Bernoulli Distribution and Binomial Distribution • Continuous Random variables, Probability Density Function, Cumulative Distribution Function

Topic 3: Expectation & Variance : • Expectation of Discrete & Continuous RVs, Expectation of general RV, Useful Properties of Expectation, Variance & Properties

Topic 4: Joint Distribution : • Joint Probability Distribution & Marginal distribution (discrete RVs), Joint Probability Distribution & Marginal distribution (continuous RVs), Expectation E[g(X,Y)] • Conditional Probability Distribution, (Statistically) Independent RV. Variance, Covariance, Correlation coefficients, Useful Results

Topic 5: Special Discrete Probability Distributions : • Binomial, Geometric (return period), Poisson Distribution, Poisson Approximation

Topic 6: Normal Distribution : • Normal Distribution, Standard Normal Distribution (introduction) • Standard Normal Distribution, Mean & Variance (Proofs excluded) • Applications: Problems, Approximation of Binomial Distribution

Topic 7: Statistics: • Population & Parameters, Samples & Sample Statistics

Topic 8: Sampling Distribution and Estimation : • Random Sampling, Sampling Distribution, Distribution of Sample Mean, Central Limit Theorem • Unbiased Estimator of variance, chi-sq- distribution, Samples with Unknown Population Variance, t-distribution • Sampling Distribution of Difference between two Means

Topic 9 Confidence Interval: I • Confidence Intervals (introduction), Known/Unknown Variance, Large/Small Sample • Confidence Intervals (1-sided, 2-sided Confidence interval), Error, Size, Prediction Interval • 2-Population: confidence Interval, Small Size Unknown Variance

Topic 10: Hypothesis Testing: • Introduction, Type-1, Type-2 Error • Hypothesis Testing for Population Mean (1-population), Power, p-value • Hypothesis Testing for Difference between Population Means Topic 11 Simple Linear Regression: • Linear Regression & Transformation • Estimation of Regression Parameters • Confidence Interval for Parameters in Linear Regression

Reading and References (if applicable)

Walpole, Myers, Myers, Ye, Probability and Statistics for Engineers and Scientists, 9th Edition, Pearson. ISBN-13:9780321629111

NOTE: The above readings comprise the foundational readings for the course and more up-to-date relevant readings will be provided when they are available.

Planned Schedule

Week or Session	Topics or Themes	ILO	Readings	Delivery Mode	Activities
1	Basic Probability Theory: • Sample space, Events, Counting Sample Points, Permutations & Combinations. • Probability, Equally Likely Outcomes, Not Equally Likely outcomes, Useful Rules • Conditional Probability, Independent Events, Partition, Bayes' Rule	1, 2	lecture notes	In-person	

Week or Session	Topics or Themes	ILO	Readings	Delivery Mode	Activities
2	Random Variables & Probability Distribution: • Random Variables (RV), Discrete Random variables, Probability Mass Distribution, Cumulative Probability Distribution • Bernoulli Distribution and Binomial Distribution • Continuous Random variables, Probability Density Function, Cumulative Distribution Function	3	lecture notes/tutorial	In-person	
3	Expectation & Variance : • Expectation of Discrete & Continuous RVs, Expectation of general RV, Useful Properties of Expectation, Variance & Properties	3, 5	lecture notes / tutorial	In-person	

Week or Session	Topics or Themes	ILO	Readings	Delivery Mode	Activities
4	Joint Distribution : • Joint Probability Distribution & Marginal distribution (discrete RVs), Joint Probability Distribution & Marginal distribution (continuous RVs), Expectation E[g(X,Y)] • Conditional Probability Distribution, (Statistically) Independent RV. Variance, Covariance, Correlation coefficients, Useful Results	4, 5	lecture notes / tutorial	In-person	
5	Special Discrete Probability Distributions : • Binomial, Geometric (return period), Poisson Distribution, Poisson Approximation	5	lecture notes / tutorial	In-person	

Week or Session	Topics or Themes	ILO	Readings	Delivery Mode	Activities
6	Normal Distribution : • Normal Distribution, Standard Normal Distribution (introduction) • Standard Normal Distribution, Mean & Variance (Proofs excluded) • Applications: Problems, Approximation of Binomial Distribution	6	lecture notes / tutorial	In-person	
7	Statistics: • Population & Parameters, Samples & Sample Statistics	7	lecture notes/tutorial	In-person	

Week or Session	Topics or Themes	ILO	Readings	Delivery Mode	Activities
8	Sampling Distribution and Estimation : • Random Sampling, Sampling Distribution, Distribution of Sample Mean, Central Limit Theorem • Unbiased Estimator of variance, chi-sq-distribution, Samples with Unknown Population Variance, t-distribution • Sampling Distribution of Difference between two Means	8, 9	lecture notes/tutorial	In-person	

Week or Session	Topics or Themes	ILO	Readings	Delivery Mode	Activities
9	Confidence Interval: • Confidence Intervals (introduction), Known/Unknown Variance, Large/Small Sample • Confidence Intervals (1-sided, 2-sided Confidence interval), Error, Size, Prediction Interval • 2-Population: confidence Interval, Small Size Unknown Variance	9	lecture notes/tutorial	In-person	
10	Confidence Interval: • 2-Population: confidence Interval, Small Size Unknown Variance	8, 9	lecture notes/tutorial	In-person	

Week or Session	Topics or Themes	ILO	Readings	Delivery Mode	Activities
11	Hypothesis Testing: • Introduction, Type-1, Type-2 Error • Hypothesis Testing for Population Mean (1-population), Power, p-value • Hypothesis Testing for Difference between Population Means	10	lecture notes/tutorial	In-person	
12	Simple Linear Regression: • Linear Regression & Transformation • Estimation of Regression Parameters • Confidence Interval for Parameters in Linear Regression	11	lecture notes/tutorial	In-person	
13	Revision	1-11	lecture notes	In-person	

Learning and Teaching Approach

Approach	How does this approach support you in achieving the learning outcomes?
Lectures	Help you understand the motivation and definitions of the concepts and notions, approaches to solving problems in pursuant to learning outcomes. LO: 1 to 11.
Tutorials	Develop problem solving skills, reinforce the understanding of the concepts and notions. LO:1 to 11

Assessment Structure

Assessment Components (includes both continuous and summative assessment)

No.	Component	ILO	Related PLO or Accreditation	Weightage	Team/Individual	Rubrics	Level of Understanding
1	Continuous Assessment (CA): Test/Quiz(Short Answer Questions 1)	1, 2, 3, 4, 5, 6	a, b, g, h, i, k	18	Individual	Analytic	Multistructural
2	Continuous Assessment (CA): Test/Quiz(Short Answer Questions 2)	7, 8, 9, 10	a, b, g, h, i, k	18	Individual	Analytic	Multistructural
3	Continuous Assessment (CA): Class Participation(Tutorial Participation)	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11	a, b, g, h, i, k	4	Individual	Analytic	Multistructural
4	Continuous Assessment (CA): Final exam(Short Answer Questions)	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11	a, b, g, h, i, k	60	Individual	Analytic	Relational

Description of Assessment Components (if applicable)

Mid-semester Quiz 1: Restricted open book quiz; Short Answer Questions; 40 minutes.
Mid-semester Quiz 2: Restricted open book quiz; Short Answer Questions; 40 minutes.
Class Participation: Attend tutorials and present the solutions for tutorial questions.
Final Exam: Restricted open book exam; Short Answer Questions; 2 hours.

Formative Feedback

Lecture: Help you understand the motivation and definitions of the concepts and notions, approaches to solving problems in pursuant to learning outcomes.
Tutorial: Develop problem solving skills, reinforce the understanding of the concepts and notions.

NTU Graduate Attributes/Competency Mapping

This course intends to develop the following graduate attributes and competencies (maximum 5 most relevant)

Attributes/Competency	Level
Decision Making	Basic
Digital Fluency	Basic
Learning Agility	Basic
Problem Solving	Intermediate
Sense Making	Basic

Course Policy

Policy (Academic Integrity)

Good academic work depends on honesty and ethical behaviour. The quality of your work as a student relies on adhering to the principles of academic integrity and to the NTU Honour Code, a set of values shared by the whole university community. Truth, Trust and Justice are at the core of NTU's shared values. As a student, it is important that you recognize your responsibilities in understanding and applying the principles of academic integrity in all the work you do at NTU. Not knowing what is involved in maintaining academic integrity does not excuse academic dishonesty. You need to actively equip yourself with strategies to avoid all forms of academic dishonesty, including plagiarism, academic fraud, collusion and cheating. If you are uncertain of the definitions of any of these terms, you should go to the academic integrity website for more information. On the use of technological tools (such as Generative AI tools), different courses / assignments have different intended learning outcomes. Students should refer to the specific assignment instructions on their use and requirements and/or consult your instructors on how you can use these tools to help your learning. Consult your instructor(s) if you need any clarification about the requirements of academic integrity in the course.

Policy (General)

You are expected to attend all classes punctually and take all scheduled tests by due dates. You are expected to take responsibility to follow up with course notes and course related announcements. You are expected to participate in all discussions and activities.

Policy (Absenteeism)

Absence from quizzes and examination without a valid reason will affect your overall course grade. Valid reasons include falling sick supported by a medical certificate and participation in NTU's approved activities supported by an excuse letter from the relevant bodies. You have to submit the Medical Certificate (or another relevant document) to the administration to obtain official leave. There are no make-up quizzes.

Policy (Others, if applicable)

Diversity and inclusion policy

Integrating a diverse set of experiences is important for a more comprehensive understanding of science.

It is our goal to create an inclusive and collaborative learning environment that supports a diversity of perspectives and learning experiences, and that honours your identities, including ethnicity, gender, socioeconomic status, sexual orientation, religion or ability.

To help accomplish this:

1. If you are neuroatypical or neurodiverse, have dyslexia or ADHD (for example), or have a social anxiety disorder or social phobia;
2. If you feel like your performance in the class is being impacted by your experiences outside of class;
3. If something was said in class (by anyone, including the instructor) that made you feel uncomfortable;

Please speak to your teaching team, our school pastoral officer or a peer or senior (either in-person or via email) about how we can help facilitate your learning experience.

As a participant in course discussions, you should also strive to honour the diversity of your classmates. You can do this by: using preferred pronouns and names; being respectful of others opinions and actively making sure all voices are being heard; and refraining from the use of derogatory or demeaning speech or actions.

All members of the class are expected to adhere to the NTU anti-harassment policy. if you witness something that goes against this or have any other concerns, please speak to your instructors or a faculty member.

Appendix 1: Assessment Rubrics

Rubric for Mid-semester Quiz: Short Answer Questions 1 (18%)

Criteria	Standards		
	Fail standard	**Pass standard**	**High standard**
Methods of approach	· Using methods that are irrelevant or do not apply to the given problem. · Invoking theorems whose conditions are not satisfied.	· Using relevant methods that help solve the problem. · Invoking theorems whose conditions are satisfied.	Finding methods and utilizing theorems that are both relevant and effective
Validity of reasoning	Reasoning is logically invalid.	Reasoning is logically valid.	Reasoning is logically valid and effective.
Clarity of argument	Reasoning is poorly explained or not explained at all.	Reasoning is clear but may contain some gaps.	Reasoning is clear, precise with no or insignificant gaps.

Rubric for Mid-semester Quiz: Short Answer Questions 2 (18%)

Criteria	Standards		
	Fail standard	**Pass standard**	**High standard**
Methods of approach	· Using methods that are irrelevant or do not apply to the given problem. · Invoking theorems whose conditions are not satisfied.	· Using relevant methods that help solve the problem. · Invoking theorems whose conditions are satisfied.	Finding methods and utilizing theorems that are both relevant and effective
Validity of reasoning	Reasoning is logically invalid.	Reasoning is logically valid.	Reasoning is logically valid and effective.
Clarity of argument	Reasoning is poorly explained or not explained at all.	Reasoning is clear but may contain some gaps.	Reasoning is clear, precise with no or insignificant gaps.

Rubric for Tutorial Participation (4%)

Criteria	Standards		
	Fail standard	**Pass standard**	**High standard**
Presentation of solution	Did not volunteer to present tutorial solution and decline to present when called.	Made one tutorial presentation during the 12 tutorial sessions.	Made two to three tutorial presentations during the 12 tutorial sessions.
Other forms of participation*	No engagement in the tutorial activities.	Engaged during tutorial to some extent, limited involvement.	Actively engaged during tutorial, such as consistently contributed to discussions, asking and answering questions.

*(Student will not be assessed on the quality of presentation and solution.)

Rubric for Examination: Short Answer Questions (60%)

Criteria	Standards		
	Fail standard	**Pass standard**	**High standard**
Methods of approach	· Using methods that are irrelevant or do not apply to the given problem. · Invoking theorems whose conditions are not satisfied.	· Using relevant methods that help solve the problem. · Invoking theorems whose conditions are satisfied.	Finding methods and utilizing theorems that are both relevant and effective

Validity of reasoning	Reasoning is logically invalid.	Reasoning is logically valid.	Reasoning is logically valid and effective.
Clarity of argument	Reasoning is poorly explained or not explained at all.	Reasoning is clear but may contain some gaps.	Reasoning is clear, precise with no or insignificant gaps.

Mapping of Course ILOs to EAB Graduate Attributes

Course Code & Title	MH2814 Probability & Statistics
Course Type	Core course

Overview											
(a)	●	(b)	●	(c)		(d)		(e)		(f)	
(g)	●	(h)	●	(i)	●	(j)		(k)	●		

Legend:

●	Fully consistent (contributes to more than 75% of Student Learning Outcome)
◐	Partially consistent (contributes to about 50% of Student Learning Outcome)
O	Weakly consistent (contributes to about 25% of Student Learning Outcome)
Blank	Not related to Student Learning Outcome

Course ILOs		**EAB Graduate Attributes**
1)	Use the techniques of permutations and combination to calculate the probability of the occurrence of an event	a, b, g, h, i, k
2)	Apply Bayes' rule to compute conditional probability and identify independent events	a, b, g, h, i, k
3)	State the definition of a random variable (RV), tabulate its probability mass function or density function, and be able to compute its expectation and variance	a, b, g, h, i, k
4)	Recognize a joint probability distribution and compute its marginal and conditional distributions	a, b, g, h, i, k
5)	Calculate the probability, mean and variance of a RV that has either binomial, geometric or Poisson distribution, and apply the Poisson approximation to binomial distribution	a, b, g, h, i, k
6)	Evaluate the probability of a normal RV, its mean and variance, and apply the normal approximation to binomial distribution	a, b, g, h, i, k
7)	Distinguish the difference between population and sample, parameter and statistic	a, b, g, h, i, k
8)	Apply the Central Limit Theorem (CLT) to the distribution of sample mean	a, b, g, h, i, k
9)	Construct confidence interval for sample mean, and determine the sample size with appropriate margin of error	a, b, g, h, i, k
10)	State the null and alternative hypotheses of a statistical test, determine the type I and type II errors, and compute its power and p-value	a, b, g, h, i, k

EAB Graduate Attributes

a) **Engineering Knowledge**: Apply the knowledge of mathematics, natural science, computing and engineering fundamentals, and an engineering specialisation as specified in WK1 to WK4 respectively to the solution of complex engineering problems.

b) **Problem Analysis**: Identify, formulate, research literature, and analyse complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences with holistic considerations for sustainable development. (WK1 to WK4)

c) **Design / Development of Solutions**: Design creative solutions for complex engineering problems and design systems, components or processes that meet identified needs with appropriate consideration for public health and safety, whole-life cost, net zero carbon as well as resource, cultural, societal, and environmental considerations as required. (WK5)

d) **Investigation**: Conduct investigations of complex problems using research-based knowledge (WK8) and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.

e) **Modern Tool Usage**: Create, select and apply appropriate techniques, resources, and modern engineering and IT tools including prediction and modelling to complex engineering problems, with an understanding of the limitations. (WK2 and WK6)

f) **The Engineer and the World**: When solving complex engineering problems, analyse and evaluate sustainable development impacts to: society, the economy, sustainability, health and safety, legal frameworks and the environment (WK1, WK5, and WK7).

g) **Ethics**: Apply ethical principles and commit to professional ethics and responsibilities and norms of engineering practice and adhere to relevant national and international laws. Demonstrate an understanding of the need for diversity and inclusion (WK9).

h) **Individual and Collaborative Team Work**: Function effectively as an individual, and as a member or leader in diverse and inclusive teams and in multidisciplinary, face-to-face, remote and distributed settings (WK9).

i) **Communication**: Communicate effectively and inclusively on complex engineering activities with the engineering community and with society at large, such as being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions, taking into account cultural, language, and learning differences.

j) **Project Management and Finance**: Demonstrate knowledge and understanding of engineering management principles and economic decision-making, and apply these to one's own work, as a member and leader in a team, to manage projects and in multidisciplinary environments.

k) **Life-long Learning**: Recognise the need for, and have the preparation and ability to (i) engage in independent and life-long learning, and (ii) adapt to new and emerging technologies, and (iii) think critically, in the broadest context of technological change (WK8).

No	Knowledge Profile
WK1	A systematic, theory-based understanding of the natural sciences applicable to the discipline and awareness of relevant social sciences
WK2	Conceptually-based mathematics, numerical analysis, data analysis, statistics and formal aspects of computer and information science to support detailed analysis and modelling applicable to the discipline
WK3	A systematic, theory-based formulation of engineering fundamentals required in the engineering discipline
WK4	Engineering specialist knowledge that provides theoretical frameworks and bodies of knowledge for the accepted practice areas in the engineering discipline; much is at the forefront of the discipline
WK5	Knowledge including efficient resource use, environmental impacts, whole-life cost, re-use of resources, net zero carbon, and similar concepts that supports engineering design and operations in a practice area
WK6	Knowledge of engineering practice (technology) in the practice areas in the engineering discipline
WK7	Knowledge of the role of engineering in society and identified issues in engineering practice in the discipline such as the professional responsibility of an engineer to public safety and sustainable development.
WK8	Engagement with selected knowledge in the current research literature of the discipline, awareness of the power of critical thinking and creative approaches to evaluate emerging issues
WK9	Knowledge of professional ethics, responsibilities, and norms of engineering practice. Awareness of the need for diversity by reason of ethnicity, gender, age, physical ability etc with mutual understanding and respect, and of inclusive attitudes

Reference: EAB Accreditation Manual