Annexe A: New/Revised Course Content in OBTL+ Format

Course Overview

The sections shown on this interface are based on the templates UG OBTL+ or PG OBTL+
If you are revising/duplicating an existing course and do not see the pre-filled contents you expect in the subsequent sections e.g. Course Aims, Intended Learning Outcomes etc. please refer to Data Transformation Status for more information.

Expected Implementation in Academic Year	AY2024-2025
Semester/Trimester/Others (specify approx. Start/End date)	Semester 1
Course Author * Faculty proposing/revising the course	Chan Song Heng
Course Author Email	chansh@ntu.edu.sg
Course Title	NUMBER THEORY
Course Code	MH3210
Academic Units	4
Contact Hours	51
Research Experience Components	Not Applicable

Course Requisites (if applicable)

Pre-requisites	MH1300
Co-requisites	
Pre-requisite to	
Mutually exclusive to	
Replacement course to	
Remarks (if any)	

Course Aims

This course aims to provide a first discovery of number theory using elementary techniques (that is techniques mostly built from scratch during the course). You will learn fundamental results on the divisibility of integers, on prime numbers, on Diophantine equations, and hear about famous conjectures in number theory. You will also practice working with congruences modulo an integer, and solving polynomial congruences and systems of linear congruences. This knowledge will be useful to you if you plan to take a course on abstract algebra, or if you are interested in applications of mathematics to cryptography.

Course's Intended Learning Outcomes (ILOs)

Upon the successful completion of this course, you (student) would be able to:

ILO 1	State fundamental results in number theory.
ILO 2	Compute modulo an integer.
ILO 3	Solve linear diophantine equations and linear congruences.
ILO 4	Prove results in number theory involving short reasoning.
ILO 5	Solve polynomial congruences and systems of linear congruences.
ILO 6	Compute primitive roots.
ILO 7	Compute Legendre symbol.

Course Content

The Euclid division algorithm and its extended version, and to use them to compute the greatest common divisor of integers. Bezout identity. The existence and unicity of writing an integer in a given basis. Fundamental Theorem of Arithmetic. Existence of an infinity of primes. Linear diophantine equations. Modular arithmetic. Linear congruences and the Chinese Remainder Theorem. Primality testing.

Reading and References (if applicable)

An introduction to the theory of numbers, by Niven, Ivan, 1915-1999.; Zuckerman, Herbert S.; Montgomery, Hugh L.

c1991; 5th ed. One book is available atLee Wee Nam Library.

Elementary Number Theory, by G. A. Jones and M. Jones, Springer Undergraduate Mathematics Series. This book is recommended and it can be accessed via NTU library. ISBN-13: 978-3540761976 ISBN-10: 3540761977 Elementary Number Theory, by Rosen. This is book contains broadly the same content, with more exercises, in particular elementary ones, however it is difficult to access it via NTU library (requires the installation of a 3rd party DRM and even after that, only 2 students at a time can access the book). ISBN-10: 129203954X ISBN-13: 978-1292039541

NOTE: The above readings comprise the foundational readings for the course and more up-to-date relevant readings will be provided when they are available.

Planned Schedule

Week or Session	Topics or Themes	ILO	Readings	Delivery Mode	Activities
1	The Euclid division algorithm, the existence and unicity of writing an integer in a given basis.	4,7	Theorem 1.10 (Rosen)or Theorem 1.1 (Jones)	In-person	
2	Existence of an infinity of primes. Bezout identity.	4,7	Theorem 3.1 (Rosen), Theorem 2.6 (Jones), Theorem 3.8 and Corollary 3.8.1 (Rosen), or Theorem 1.7 (Jones)	In-person	
3	The extended Euclid algorithm, and to use them to compute the greatest common divisor of integers. Fundamental Theorem of arithmetic.	4,7	Theorem 3.14 (Rosen) or Theorem 1.6 (Jones), Theorem 3.15 (Rosen)or Theorem 2.3 (Jones)	In-person	
4	Modular arithmetic, modular exponentiation, Linear diophantine equations.	5,6	Theorem 3.23 and 3.24 (Rosen) or Theorem 3,7 (Jones), Theorem 4.1, 4.2 (Rosen)or Lemma 3.1 (Jones)	In-person	
5	Linear congruences and the Chinese Remainder Theorem.	3,5,6	Theorem 4.11 (Rosen)or Theorem 3.7 (Jones), Theorem 4.12, Theorem 4.13 (Rosen)or Theorem 3.10 (Jones)	In-person	
6	Euler function, and Euler's and Wilson's theorems	4,5	Theorem 5.3 (Jones), Theorem 6.13 (Rosen), Corollary 4.5 (Jones), Theorem 6.1 (Rosen)	In-person	Mid-Semester Test 1

Week or Session	Topics or Themes	ILO	Readings	Delivery Mode	Activities
7	Applications of Euler's Theorem, e.g. RSA	4,5	section 8.4 p. 323-324 by Rosen or p. 95 by Jones	In-person	
8	Existence of primitive roots	2,5	Theorems 6.5, 6.7, 6.11 (Jones), Theorems 9.1, 9.8, 9.10, 9.15 (Rosen)	In-person	
9	Applications of primitive roots	2,5	section 6.6 (Jones), chapter 10 (Rosen)	In-person	
10	Quadratic residues and Legendre symbol	1,5	chapter 11 (Rosen), chapter 7 (Jones)	In-person	Mid-Semester Test 2
11	Legendre symbol and quadratic reciprocity	1,4,5	chapter 11 (Rosen), chapter 7 (Jones)	In-person	
12	Jacobi symbol	1,5	chapter 11 (Rosen), chapter 7 (Jones)	In-person	
13	Pythagorean triples	4	chapter 11 (Jones), section 13.1 (Rosen)	In-person	

Learning and Teaching Approach

Approach	How does this approach support you in achieving the learning outcomes?
Lectures	Proofs (e.g. for proving divisibility properties of integers) are done on the board, this is to make sure that the pace is appropriate and you get to see how every step is done. Small exercises are provided during the lecture, to be discussed in groups, to make sure that you have understood the new topic/definition before moving on. This also gives you the opportunity to practice computations in class (e.g. modular arithmetic, solving linear equations). Whenever possible, plots/animations will be provided to give you a visualization of abstract functions/concepts (e.g. fundamental results in number theory).
Tutorials	Exercises will belong to typically two categories: small proofs, so you get trained in formulating and proving results in number theory (e.g. for providing divisibility properties of integers), and computations, so you develop the skills to be able to work with modular arithmetic, solve linear integer/congruence equations, systems of congruences.

Assessment Structure

Assessment Components (includes both continuous and summative assessment)

No.	Component	ILO	Related PLO or Accreditation	Weightage	Team/Individual	Rubrics	Level of Understanding
1	Continuous Assessment (CA): Test/Quiz(Short Answer Questions 1)	2, 3, 4, 5	Not Applicable	15	Individual	Analytic	Multistructural
2	Continuous Assessment (CA): Test/Quiz(Short Answer Questions 2)	4, 5, 6, 7	Not Applicable	15	Individual	Analytic	Multistructural
3	Continuous Assessment (CA): Assignment(Short Answers, 3 homework sets)	1, 2, 3, 4, 5, 6	Not Applicable	6	Individual	Analytic	Multistructural
4	Continuous Assessment (CA): Class Participation(Tutorial presentation and participation)	1, 2, 3, 4, 5, 6, 7	Not Applicable	4	Individual	Holistic	Multistructural
5	Summative Assessment (EXAM): Final exam(Short Answer Questions)	1, 2, 3, 4, 5, 6, 7	Not Applicable	60	Individual	Analytic	Multistructural

Description of Assessment Components (if applicable)

For Continuous Assessment (CA): Class Participation(Tutorial presentation and participation): You are required to present at least twice during the 12 tutorial sessions in the semester.

Formative Feedback

You will have the opportunity to discuss your understanding of results in number theory, of the techniques taught to execute different types of computations (modulo an integer, solving linear equations) and to prove results (a number of them related to divisibility properties) during lectures via group discussions, during which feedback will be provided by peers and the lecturer.
You will also have the option to present solutions of your exercises during the tutorials, which will receive feedback from the lecturer.

Midterm assessments will be graded and feedback will be provided for each student on the area(s) that should be improved (if any) and those which are already satisfactory.
After the exam period, a feedback on the final exam will be uploaded on NTULearn.

NTU Graduate Attributes/Competency Mapping

This course intends to develop the following graduate attributes and competencies (maximum 5 most relevant)

Attributes/Competency	Level
Creative Thinking	Basic
Curiosity	Basic
Problem Solving	Advanced
Sense Making	Intermediate
Transdisciplinarity	Basic

Course Policy

Policy (Academic Integrity)

Good academic work depends on honesty and ethical behaviour. The quality of your work as a student relies on adhering to the principles of academic integrity and to the NTU Honour Code, a set of values shared by the whole university community. Truth, Trust and Justice are at the core of NTU's shared values. As a student, it is important that you recognize your responsibilities in understanding and applying the principles of academic integrity in all the work you do at NTU. Not knowing what is involved in maintaining academic integrity does not excuse academic dishonesty. You need to actively equip yourself with strategies to avoid all forms of academic dishonesty, including plagiarism, academic fraud, collusion and cheating. If you are uncertain of the definitions of any of these terms, you should go to the academic integrity website for more information. On the use of technological tools (such as Generative AI tools), different courses / assignments have different intended learning outcomes. Students should refer to the specific assignment instructions on their use and requirements and/or consult your instructors on how you can use these tools to help your learning. Consult your instructor(s) if you need any clarification about the requirements of academic integrity in the course.

Policy (General)

As for every course, please try to be as ready as possible before coming to class. This means you should have tried to solve the tutorial exercises, and reminded yourself of the topic taught.

Policy (Absenteeism)

You are expected to attend the midterm tests. A student who is absent from midterm test without valid Leave of Absence will be given zero mark. No makeup midterm test will be arranged. In case of valid reason for absence, the total course marks would subsequently be rescaled to a base of 100%.

Policy (Others, if applicable)

Diversity and inclusion policy
Integrating a diverse set of experiences is important for a more comprehensive understanding of science.
It is our goal to create an inclusive and collaborative learning environment that supports a diversity of perspectives and learning experiences, and that honours your identities; including ethnicity, gender, socioeconomic status, sexual orientation, religion or ability.
To help accomplish this:

- If you are neuroatypical or neurodiverse, have dyslexia or ADHD (for example), or have a social anxiety disorder or social phobia;
- If you feel like your performance in the class is being impacted by your experiences outside of class;
- If something was said in class (by anyone, including the instructor) that made you feel uncomfortable;

Please speak to your teaching team, our school pastoral officer or a peer or senior (either in-person or via email) about how we can help facilitate your learning experience.
As a participant in course discussions, you should also strive to honour the diversity of your classmates. You can do this by: using preferred pronouns and names; being respectful of others opinions and actively making sure all voices are being heard; and refraining from the use of derogatory or demeaning speech or actions.
All members of the class are expected to adhere to the NTU anti-harassment policy. if you witness something that goes against this or have any other concerns, please speak to your instructors or a faculty member.

Appendix 1: Assessment Rubrics

Rubric for Mid-semester Quiz: Short Answer Questions 1 (15%)

The assessment comprise short answer questions, of two types: short reasoning and computations.

Assessment criteria include:

right answer,

proper arguments and justifications,

details provided.

To score a high mark, it is needed that the right answer is provided with a justification. It is also possible to have a high score while makin g some small mistake, if the argumentation is clear and the mistake is coming from for example a typo, or an inattention mistake.

A right answer with no justification whatsoever will result in a pass mark, enough evidence of understanding even with a wrong answer will result in a pass mark.

A wrong answer with no argument, or a wrong answer with wrong justification, or no answer will result in a fail mark.

Rubric for Mid-semester Quiz: Short Answer Questions 2 (15%)

The assessment comprise short answer questions, of two types: short reasoning and computations.

Assessment criteria include:

right answer,

proper arguments and justifications,

details provided.

To score a high mark, it is needed that the right answer is provided with a justification. It is also possible to have a high score while makin g some small mistake, if the argumentation is clear and the mistake is coming from for example a typo, or an inattention mistake.

A right answer with no justification whatsoever will result in a pass mark, enough evidence of understanding even with a wrong answer will result in a pass mark.

A wrong answer with no argument, or a wrong answer with wrong justification, or no answer will result in a fail mark.

Rubric for Examination: Short Answer Questions (60%)

The assessment comprise short answer questions, of two types: short reasoning and computations.

Assessment criteria include:

right answer,

proper arguments and justifications,

details provided.

To score a high mark, it is needed that the right answer is provided with a justification. It is also possible to have a high score while makin g some small mistake, if the argumentation is clear and the mistake is coming from for example a typo, or an inattention mistake.

A right answer with no justification whatsoever will result in a pass mark, enough evidence of understanding even with a wrong answer will result in a pass mark.

A wrong answer with no argument, or a wrong answer with wrong justification, or no answer will result in a fail mark.

Rubric for Assignment: Short Answer Questions 2 (6%)

The assessment comprise short answer questions, of two types: short reasoning and computations.

Assessment criteria include:

right answer,

proper arguments and justifications,

details provided.

To score a high mark, it is needed that the right answer is provided with a justification. It is also possible to have a high score while makin g some small mistake, if the argumentation is clear and the mistake is coming from for example a typo, or an inattention mistake.

A right answer with no justification whatsoever will result in a pass mark, enough evidence of understanding even with a wrong answer will result in a pass mark.

A wrong answer with no argument, or a wrong answer with wrong justification, or no answer will result in a fail mark.

Rubric for Class Participation: Tutorial Presentation and Participation (4%)

The assessment comprise short answer questions, of two types: short reasoning and computations.

Assessment criteria include:

right answer,

proper arguments and justifications,

details provided,

Clarity of presentation,

Engagement and Interaction.

To score a high mark, it is needed that the right answer is provided with a justification. It is also possible to have a high score while makin g some small mistake, if the argumentation is clear and the mistake is coming from for example a typo, or an inattention mistake. The pres entation must be clear and logical with effective use of the board and engages the audience throughout.

A right answer with no justification whatsoever will result in a pass mark, enough evidence of understanding even with a wrong answer will result in a pass mark. The presentation is somewhat clear but there are noticeable issues in logic or board usage with limited audience en gagement.

A wrong answer with no argument, or a wrong answer with wrong justification, or no answer will result in a fail mark. Presentation is uncle ar and disorganized and there is no engagement with audience.