Annexe A: New/Revised Course Content in OBTL+ Format

Course Overview

The sections shown on this interface are based on the templates UG OBTL+ or PG OBTL+
If you are revising/duplicating an existing course and do not see the pre-filled contents you expect in the subsequent sections e.g. Course Aims, Intended Learning Outcomes etc. please refer to Data Transformation Status for more information.

Expected Implementation in Academic Year	AY2024-2025
Semester/Trimester/Others (specify approx. Start/End date)	Semester 2
Course Author * Faculty proposing/revising the course	Andrew James Kricker
Course Author Email	ajkricker@ntu.edu.sg
Course Title	ALGEBRAIC TOPOLOGY
Course Code	MH4600
Academic Units	4
Contact Hours	51
Research Experience Components	Not Applicable

Course Requisites (if applicable)

Pre-requisites	“MH3200 and MH3600” or “MH2220 and MH3600”
Co-requisites	
Pre-requisite to	
Mutually exclusive to	
Replacement course to	
Remarks (if any)	

Course Aims

This course will introduce the point of view, framework and most important tools of Algebraic Topology. Algebraic Topology is the mathematical theory whose fundamental problem is the investigation of topological spaces and related concepts using tools from abstract algebra. The ideas and tools from Algebraic Topology are important in many parts of pure mathematics and are becoming increasingly important in physics and in data science.
The course aims to give you a foundational understanding in the two most important topics within Algebraic Topology.
1. The theory of homology.
2. The theory of fundamental group and covering spaces.
The aim is for you to be sufficiently prepared to continue deeper study in this topic, perhaps at the graduate level. The aim is also to equip you so that when you encounter these ideas in different topics (such as in physics or in data science) then you have the ability to bring in an expert understanding of the theory and the ability to deepen your learning as is needed in the context.

Course's Intended Learning Outcomes (ILOs)

Upon the successful completion of this course, you (student) would be able to:

ILO 1	Construct spaces using simplicial or CW complexes, determine the corresponding chain complexes, and compute corresponding algebraic topological quantities such as homology and cohomology groups.
ILO 2	Prove algebraic properties about homology groups by using tools from the theory of homological algebra, such as exact sequences and diagram chasing techniques.
ILO 3	Define the fundamental group of a space and develop its fundamental properties.
ILO 4	Classify the system of covering spaces of a topological space in terms of the system of subgroups of the fundamental group of the space.
ILO 5	Formulate mathematical ideas and relationships using the language of category theory.
ILO 6	Apply your expert background in algebraic topology to be able to engage with studies in other fields, such as physics and data science, which exploit concepts from algebraic topology.

Course Content

- Introduction - The motivations for the theory of algebraic topology, the language of category theory and some fundamental examples, such as the Brouwer fixed point theorem.
- Simplices and simplicial complexes - The definition of an n-simplex and related concepts suchas the faces of a simplex, barycentric coordinates and barycentric subdivision, the definition of a simplicial complex and examples of simplicial complexes.
- Chain complexes and their homology groups - The chain complex associated to a simplicial complex and its associated boundary map, the definitions of cycles, boundaries, and homologygroups of chain complexes.
- Homological algebra - Chain homotopies, short and long exact sequences, axioms of a homology theory and alternative theories of homology.
- Fundamental group - Concept of homotopy, and the definition and fundamental properties ofthe fundamental group functor.
- Covering spaces - The fundamental group of the circle, the definition of a covering map, lifting paths and homotopies to covering spaces.
- Covering spaces and the fundamental group - The fundamental group of a covering space embeds in the fundamental group of the base space, and lifting maps to covering spaces.
- The fundamental theorem of covering spaces - Isometries of covering maps, spaces constructed as quotients by group actions, construction of the universal cover of a space, normal subgroups and normal covers.

Reading and References (if applicable)

Kosniowski, C., A first course in algebraic topology, Cambridge University Press. ISBN 978-0-521-29864-3

Munkres J.R., Elements of algebraic topology, Perseus Publishing. ISBN: 0-201-62728-0

Hatcher A., Algebraic Topology, Cambridge University Press. ISBN 978-0-521-79540-1

Planned Schedule

Week or Session	Topics or Themes	ILO	Readings	Delivery Mode	Activities
1	Introduction to the purpose of the theory of algebraic topology. The language of category theory. Some basic examples.	5, 6	Study lecture notes.	In-person	Solve problems.
2	Theory of simplices. Definition of simplicial complexes. Examples of simplicial complexes.	1	Study lecture notes.	In-person	Solve problems. Presentations of solutions and theory.
3	Definition of chain groups of a complex. The boundary map. The chain complex of a simplicial complex. Definitions of cycles, boundaries, and homology groups.	1, 2	Study lecture notes.	In-person	Solve problems. Presentations of solutions and theory.

Week or Session	Topics or Themes	ILO	Readings	Delivery Mode	Activities
4	Definition of chain groups of a complex. The boundary map. The chain complex of a simplicial complex. Definitions of cycles, boundaries, and homology groups.	1, 2	Study lecture notes.	In-person	Solve problems. Presentations of solutions and theory.
5	Algorithms to compute homology groups. Examples of computations.	1, 2	Study lecture notes.	In-person	Solve problems. Presentations of solutions and theory.
6	Definition of relative homology. Simplicial maps. Chain maps. Induced homomorphisms of homology groups.	1, 2	Study lecture notes.	In-person	Solve problems. Presentations of solutions and theory.
7	Brief survey of singular homology groups. Statement of homotopy invariance property. Axioms of a homology theory.	1, 2	Study lecture notes.	In-person	Solve problems. Presentations of solutions and theory.

Week or Session	Topics or Themes	ILO	Readings	Delivery Mode	Activities
8	Fundamental group. Definition as a functor. Homotopy invariance.	3, 5	Study lecture notes.	In-person	Solve problems. Presentations of solutions and theory.
9	Introduction to the theory of covering spaces by the computation of the fundamental group of the circle. Definition of a covering map. Fundamental development of the theory including lemmas about the existence of lifts of paths and homotopies.	4	Study lecture notes.	In-person	Solve problems. Presentations of solutions and theory.
10	Introduction to the theory of covering spaces by the computation of the fundamental group of the circle. Definition of a covering map. Fundamental development of the theory including lemmas about the existence of lifts of paths and homotopies.	4	Study lecture notes.	In-person	solve problems. Presentations of solutions and theory.

Week or Session	Topics or Themes	ILO	Readings	Delivery Mode	Activities
11	The theory that classifies covering spaces of a fixed space in terms of subgroups of the fundamental group of the space. Key points include: characterization of covering spaces in terms of subgroups. Lifting theorem for general maps into the base space. Isometries of the covers. Quotient spaces by group actions. Construction of the universal covering space.	4	Study lecture notes.	In-person	Solve problems. Presentations of solutions and theory.

Week or Session	Topics or Themes	ILO	Readings	Delivery Mode	Activities
12	The theory that classifies covering spaces of a fixed space in terms of subgroups of the fundamental group of the space. Key points include: characterization of covering spaces in terms of subgroups. Lifting theorem for general maps into the base space. Isometries of the covers. Quotient spaces by group actions. Construction of the universal covering space.	4	Study lecture notes.	In-person	Solve problems. Presentations of solutions and theory.
13	Student presentations of their projects.	1-6		In-person	Project presentations.

Learning and Teaching Approach

Approach	How does this approach support you in achieving the learning outcomes?
Lectures	Present the key ideas behind mathematical concepts. Present important steps used to solve different types of problems.
Tutorials	Develop proficiency in problem solving skills. Reinforce concepts already covered in the lectures. Give an opportunity for weaker or more reserved students to clarify doubts.

Assessment Structure

Assessment Components (includes both continuous and summative assessment)

No.	Component	ILO	Related PLO or Accreditation	Weightage	Team/Individual	Rubrics	Level of Understanding
1	Continuous Assessment (CA): Assignment(Students solve problems applying and expanding the course materials including problems sourced by the lecturer as well as problems selected by the students.)	1, 2, 3, 4, 5		20	Individual	Analytic	Extended Abstract
2	Continuous Assessment (CA): Presentation(Students present their solutions to the class as well as provide short talks on designated topics to supplement the lectures)	1, 2, 3, 4, 5		10	Individual	Analytic	Extended Abstract
3	Continuous Assessment (CA): Project(Learning project. Students independently choose a topic which expands the course material and give a short talk to the class teaching them about this topic.)	1, 2, 3, 4, 5, 6		10	Individual	Analytic	Extended Abstract
4	Summative Assessment (EXAM): Final exam(Final exam.)	1, 2, 3, 4, 5		60	Individual	Analytic	Extended Abstract

Description of Assessment Components (if applicable)

Continuous Assessment (CA): Assignment (Students solve problems applying and expanding the course materials including problems sourced by the lecturer as well as problems selected by the students.)

Continuous Assessment (CA): Presentation (Students present their solutions to the class as well as provide short talks on designated topics to supplement the lectures.)

Continuous Assessment (CA): Project (Learning project. Students independently choose a topic which expands the course material and give a short talk to the class teaching them about this topic.)

Summative Assessment (EXAM): Final exam (Final Examination)

Formative Feedback

Feedback on common mistakes and the level of difficulty of the homework problems is given. Feedback on performance in the group project will also be given to each student.

NTU Graduate Attributes/Competency Mapping

This course intends to develop the following graduate attributes and competencies (maximum 5 most relevant)

Attributes/Competency	Level
Collaboration	Basic
Communication	Advanced
Curiosity	Advanced
Problem Solving	Advanced
Transdisciplinarity	Intermediate

Course Policy

Policy (Academic Integrity)

Good academic work depends on honesty and ethical behaviour. The quality of your work as a student relies on adhering to the principles of academic integrity and to the NTU Honour Code, a set of values shared by the whole university community. Truth, Trust and Justice are at the core of NTU's shared values. As a student, it is important that you recognize your responsibilities in understanding and applying the principles of academic integrity in all the work you do at NTU. Not knowing what is involved in maintaining academic integrity does not excuse academic dishonesty. You need to actively equip yourself with strategies to avoid all forms of academic dishonesty, including plagiarism, academic fraud, collusion and cheating. If you are uncertain of the definitions of any of these terms, you should go to the academic integrity website for more information. On the use of technological tools (such as Generative AI tools), different courses / assignments have different intended learning outcomes. Students should refer to the specific assignment instructions on their use and requirements and/or consult your instructors on how you can use these tools to help your learning. Consult your instructor(s) if you need any clarification about the requirements of academic integrity in the course.

Policy (General)

Required policy is discussed in Academic Integrity and Absenteeism sections.

Policy (Absenteeism)

Absence Due to Medical or Other Reasons

If you are sick and not able to attend in a class you are scheduled to give a presentation, you have to submit the original Medical Certificate (or another relevant document) to the administration to obtain official leave. In this case, the missed presentation will be rescheduled for another class.

Policy (Others, if applicable)

Diversity and inclusion policy

Integrating a diverse set of experiences is important for a more comprehensive understanding of science.

It is our goal to create an inclusive and collaborative learning environment that supports a diversity of perspectives and learning experiences, and that honours your identities; including ethnicity, gender, socioeconomic status, sexual orientation, religion or ability.

To help accomplish this:

If you are neuroatypical or neurodiverse, have dyslexia or ADHD (for example), or have a social anxiety disorder or social phobia;
If you feel like your performance in the class is being impacted by your experiences outside of class; If something was said in class (by anyone, including the instructor) that made you feel uncomfortable;

Please speak to your teaching team, our school pastoral officer or a peer or senior (either in-person or via email) about how we can help facilitate your learning experience.

As a participant in course discussions, you should also strive to honour the diversity of your classmates. You can do this by: using preferred pronouns and names; being respectful of others opinions and actively making sure all voices are being heard; and refraining from the use of derogatory or demeaning speech or actions.

All members of the class are expected to adhere to the NTU anti-harassment policy. if you witness something that goes against this or have any other concerns, please speak to your instructors or a faculty member.

Appendix 1: Assessment Rubrics

Rubric for Tutorials: Assignment (20%)

Point-based marking (not rubrics based)

Rubric for Tutorials: Presentation (10%)

Each student has to present 5 times during the semester, and is graded individually in the style of an oral examination.

Grading Criteria	Exceptional (17-20)	Effective (14-16)	Acceptable (10-13)	Developing (0-9)
Accuracy	The interpretation is highly accurate, concise and precise.	The interpretation is mostly accurate. Some parts can be better explained or more succinct.	The interpretation is somewhat accurate. However, it contains some inaccuracies, missing points or ideas that are not related to the interpretation.	The interpretation are mostly inaccurate.
Thoroughness	The literature review was comprehensive and rigorous. It includes several different perspectives, including a good spread of the first and latest ideas on the topic.	The literature review was mostly comprehensive and rigorous. It can improve in terms of the selection of the works relating to the topic.	The literature review was adequate. It covers some of the major works relating to the topic. References to primary source is largely missing.	The literature review was not thorough. It is based on a single source of information and/or inaccurate or unreliable secondary sources.
Presentation	Very clear and organized. It is easy to follow your train of thought	Mostly clear and organized. Some parts can have better transitions.	Somewhat clear. It requires some careful reading to understand what you are writing.	Mostly unclear and messy. It is difficult to understand what you are writing as there is no

				clear flow of ideas.
Question and Answer (for each individual student)	Very clear and precise answers to all problems. Explain the problems from various different perspectives logically.	Correct answers to most of the problems. Explain the problems in an organized way.	Partially-correct answers to most of the problems. Explain the some of the problems .	Unclear and messy answers. Difficult to understand.

Rubric for Tutorials: Project (10%)

Students are graded individually and will have to give a final presentation in the last week of class.

Grading Criteria	Exceptional (17-20)	Effective (14-16)	Acceptable (10-13)	Developing (0-9)
Accuracy	The interpretation is highly accurate, concise and precise.	The interpretation is mostly accurate. Some parts can be better explained or more succinct.	The interpretation is somewhat accurate. However, it contains some inaccuracies, missing points or ideas that are not related to the interpretation.	The interpretation are mostly inaccurate.
Thoroughness	The literature review was comprehensive and rigorous. It includes several different perspectives, including a good spread of the first and latest ideas on the topic.	The literature review was mostly comprehensive and rigorous. It can improve in terms of the selection of the works relating to the topic.	The literature review was adequate. It covers some of the major works relating to the topic. References to primary source is largely missing.	The literature review was not thorough. It is based on a single source of information and/or inaccurate or unreliable secondary sources.

Presentation	Very clear and organized. It is easy to follow your train of thought	Mostly clear and organized. Some parts can have better transitions.	Somewhat clear. It requires some careful reading to understand what you are writing.	Mostly unclear and messy. It is difficult to understand what you are writing as there is no clear flow of ideas.
Originality	Evidence of extensive synthesis of ideas from different perspectives such that there is a very convincing original interpretation and that goes beyond what is already discuss	Evidence of some synthesis of ideas which lead to an original interpretation. The interpretation is good original summary of what is discussed in literature.	Evidence of an attempt to synthesise ideas. However, the attempt contains some misunderstandin gs.	No synthesis of ideas or originality. It is a repetition of what people have said or a laundry list of ideas with little interpretation.

Rubric for Examination: Final Examination (60%)

Point-based marking (not rubrics based)