Teaching and Learning through C-P-A Sequence

MME Graduate Seminar
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27 November 2015

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Overview of Theoretical Framework

Dienes *(Dinenes, 1971)*

Inter-action, Rule construction and manipulation, isomorphisms, representation, symbolization, formalization

"…mathematics is an abstract subject. It follows that most people who have studied the problem of learning such an abstract subject, would agree that some passage from the concrete to the abstract must be mapped."

Bruner *(Bruner & Kenney, 1965)*

Enactment, Iconic & Symbolic

"The problem sequences were designed to provide, first, an appreciation of mathematical ideas through concrete constructions using materials of various kinds for these constructions. From such constructions, the child was encouraged to form perceptual images of the mathematical idea in terms of the forms that had been constructed."

Piaget *(Ginsburg & Opper, 1988)*

Concrete Operational

"…the essence of knowledge is activity."

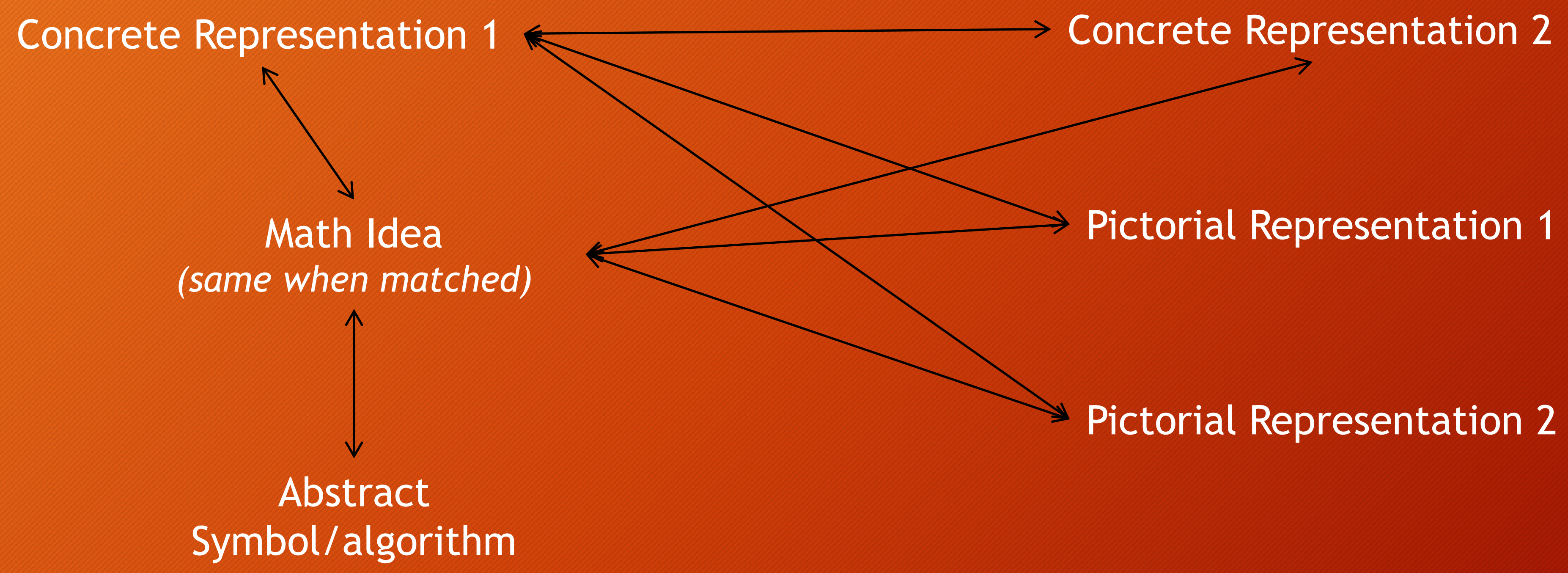
"If the child's physical actions are not accompanied by parallel mental activity, such as thinking of alternatives types of results and their meaning, it is unlikely that much real and lasting learning will occur."

Representations

(Abstractions, internal and external representations)

Teaching and Learning as an integrated unit in mathematical conceptual acquisition

Diagrammatic depiction of Model



Proposed Theoretical Teaching and Learning Model

Learning Phases		Teaching Facilitations	
		<u>Key Features</u>	<u>Pointers to teachers</u>
1	Initial Guided Explication *Identification of math idea in representation*	Presentation of representation for manipulation	• Highlight math idea embedded in the representations • Ensure learner knows the math idea
2	Exploratory Familiarization *Formation of mental nodes of math idea and representation. Weak connections are formed.*	Provision of opportunities for manipulation of representation	• Reiterate math idea embedded in representation • Ensure learner familiar with math idea and representation
3	Classification *Connections of math idea and representation fully formed and connected.*	Provision of a variety of questions to strengthen the connections of representations and math idea	• Highlight how math idea is related to representation • Ensure learner make connections between math idea and representation
4	Reification *Dominant use of symbols with occasional point of reference to representation.*	Reduce reliance on representation, use mostly math symbol	• Encourage learner to work with, reason and communicate primarily with math symbol

Implementation Background

Intervention Programme

Low Progress P3 students on the concept of equivalent fractions.
A series of lesson plans crafted using the model.

Whole-class Approach

Week 1 – Level Up
Week 2 to 4 – Equivalent Fractions
n = 25

After-school Programme (Remediation) Approach

1h x 2 days x 1 week (Level up) +
1h x 2 days x 3 weeks
n = 26

Data Collected

Interview (Task-based, Semi-structured)	
1	Pre-intervention
2	After introduction of equivalent fractions
3	After introduction of ‘short-cut’ algorithm

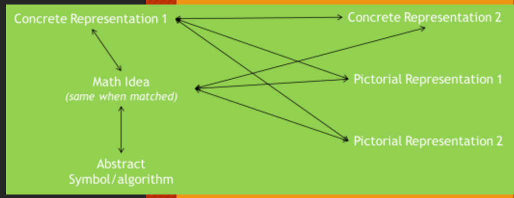
Classroom Video-Clips (teacher and pupils)

Students’ Work (worksheets)

Test 1 and Test 2

Interview setting & External Representations employed

Connections manifested in behavioural terms



Concrete Representation (Fraction discs/strips)	**Pictorial** Representation (Circular/Rectangular Model)	After matching the concrete/pictorial representations, area model of given fractions depicted are of the same size (Math Idea)
Abstract (Symbol)	**Concrete** Representation	a/b=c/d is explained through the use of concrete representations to depict area models of given fractions are of the same size
Abstract (Symbol)	**Pictorial** Representation	a/b=c/d is explained through the use of pictorial representations to depict area model(s) of given fractions are of the same size
Abstract (Algorithm)	**Concrete** Representation	Relational understanding demonstrated through concrete representation (partition of parts)
Abstract (Algorithm)	**Pictorial** Representation	Relational understanding demonstrated through pictorial representation (partition of parts)

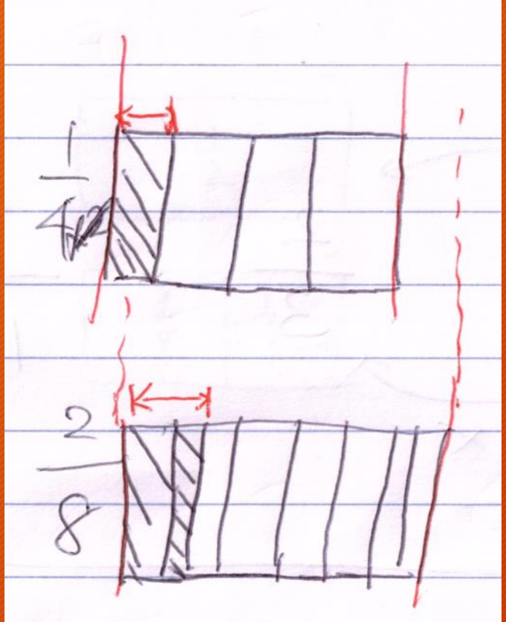
Findings – 3 case studies

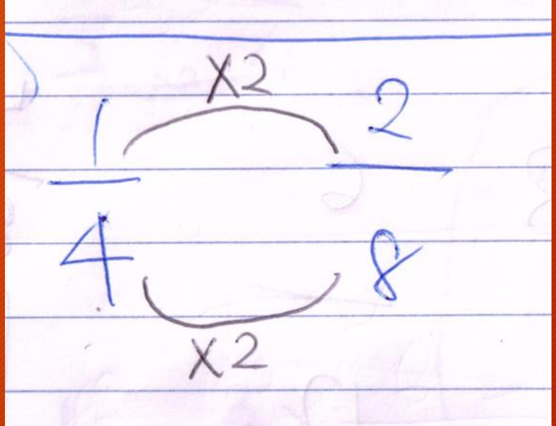
- Mental nodes formed but with no connections
- Mental nodes formed but with connections primarily only at concrete and pictorial level
- Mental nodes formed with desired connections made with teacher facilitation

Case Study 1 *(RemBS13)*

No connections

- RBS13 matched the concrete fraction discs of one-quarter and two-eighths but concluded that the latter is bigger than the former as it requires two 'pieces' (eighths) for matching





Concrete Representation

Math Idea

Pictorial Representation

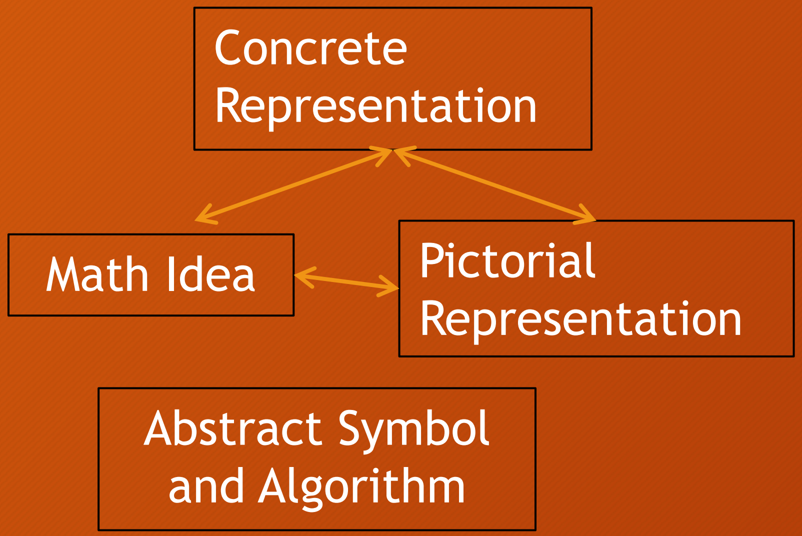
Abstract Symbol and Algorithm

Case Study 2 (WCS26)

Connections primarily only at concrete and pictorial level

Interview snippet:

T	What do you mean when you wrote 2/3=4/6?
WS26	(silence)
T	Why do you put the equal sign?
WS26	Because they are equal.
T	What do you mean by they are equal?
WS26	They are equivalent fraction.
T	What do you mean by equivalent fractions?
WS26	That means same denominator.



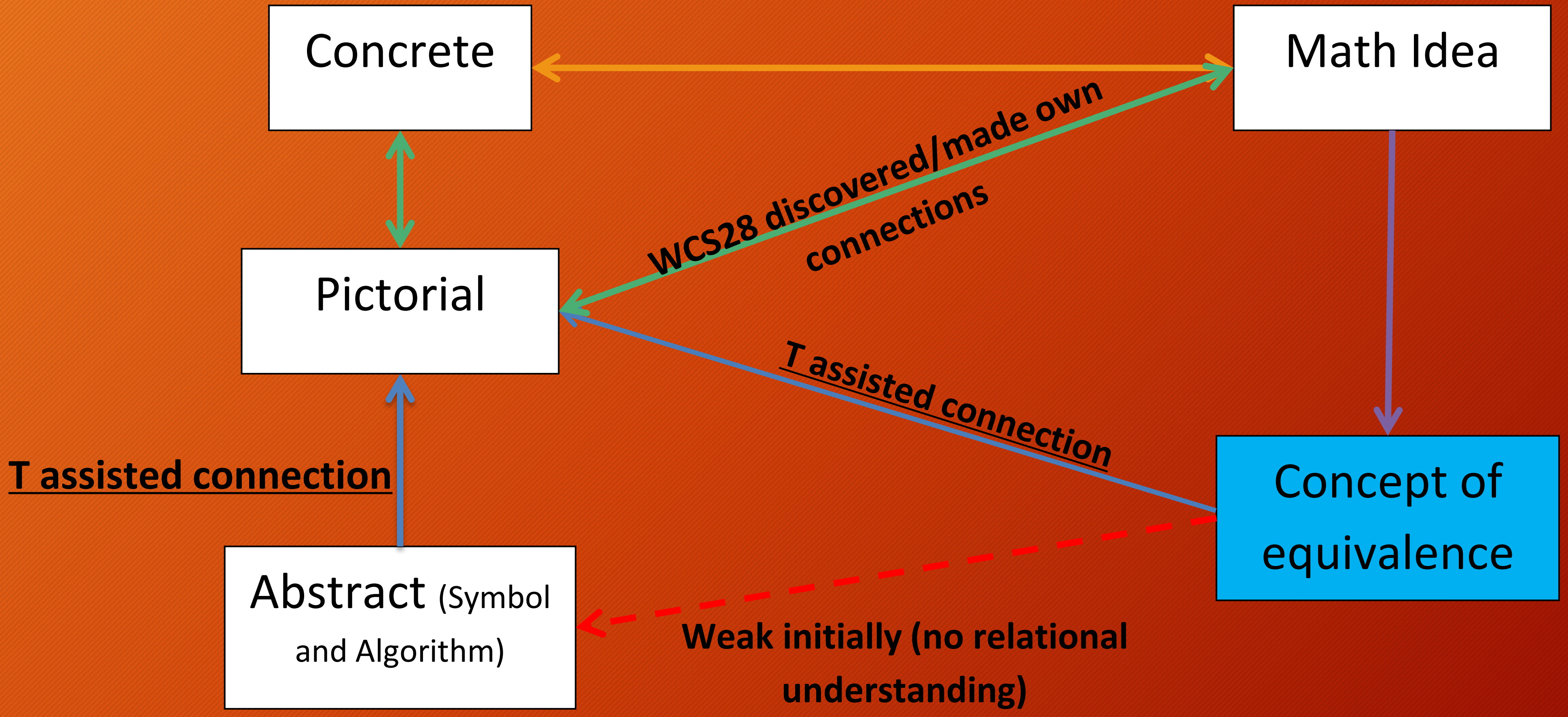
Case Study 3 (WCS28)

Connections facilitated by teacher cum interviewer

T	Okay, two-thirds is this. Can you show me four-sixths?
WS28	(figuring out what to do)
T	Can you draw four-sixths inside here (pointed to rectangular bar of 2/3)? Or must draw a separate one?
WS28	Separate one.
T	Why must it be separate? Cannot be inside?
WS28	(felt awkward, shrugged her shoulders) I don't know.
T	But when you multiply by two, what will you do to the picture?
WS28	Make the picture into smaller parts.
T	So let's say we use this (pointed to last rectangular block drawn for 2/3). This is two-thirds, right? We multiply by two, what do you do to the picture?
WS28	Cut each part into two.
T	Can you do that for me?
WS28	(sub-partitioned each part into two) Ah, this thing show four out of six. This one four pieces I shaded.

Case Study 3 (WCS28)

Connections facilitated by teacher cum interviewer



Implications and Reflections

Case Study	Implications	Reflections
(1) No connections	Provision of opportunities for manipulation is insufficient in concept acquisition. Actions undertaken can be meaningless. Enactment and Engagement is only a small part of the learning equation.	During the learning phase “Exploratory Familiarization” and “Classification”, teacher facilitation needs to include opportunities for i) (Teacher-Directed) Self-reflection ii) (Teacher-Directed) Accommodation & Assimilation
(2) Connections primarily at concrete, pictorial and math idea only (3) teacher facilitated connections	Ability to associate math idea (sameness) with representations (concrete or pictorial) is insufficient in deep learning. Learning remains only at superficial level. Process of deeper reflection is necessary for math idea to be connected to selected concept.	

Thank You