Examples in the Teaching of Mathematics: Teachers' Perceptions

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Contents

Background of the Problem

The main study aims to examine how

- Mathematics teachers in **Singapore**, select and implement instructional examples in their secondary classrooms.
- Teachers' knowledge shape their use of instructional examples in the teaching of mathematics and
- Teachers' use of instructional examples expands and refines their mathematical knowledge for teaching

Instructional Example

"Any example offered by either a teacher or a student within the context of learning a particular topic"

(Zodik & Zaslavsky, 2008, p.166)

Rationale of Study

Examples In Mathematics Instruction

- Pervasive and Established
- Essential for concept formation and teaching of procedures

  "The raw material for generalizing processes and conceptualizing new objects" (Bills et al., 2006, p. 127)

  "We teach a (general) procedure by a (particular) performance of that procedure". (Rowland, Thwaites, & Huckstep, 2003, p. 4)

Lack of Attention on Instructional Examples

- Volume of research remains little (Bills et al., 2006; Chick, 2007)
- Research in exemplification is unexplored in Singapore

Connectedness between Examples and Teacher Knowledge

- Teachers' example choice is an indicator of their knowledge (Chick, 2007; Rowland et al., 2005)
- The ability to convert tasks into effective lessons relies on teachers' PCK (Sullivan, Clarke, Clarke, & O'Shea, 2009)
- Both prospective (Rowland, 2008) and experienced teachers (Zodik & Zaslavsky, 2008) face problems
- "Even well-chosen examples are not necessarily easy to implement effectively in the classroom" (Chick & Pierce, 2008, p. 321).

Literature

Teacher Knowledge

"Mathematical Knowledge entailed by teaching" (Ball, Thames, & Phelps, 2008, p.399)

Content Knowledge of Mathematics

Pedagogical Content Knowledge

Craft Knowledge

Framework of Teacher Knowledge (Ball, Thames, & Phelps, 2008)

Connectedness

Three Studies with Elementary Teachers

Choice and Use of Examples related to Teachers' PCK and Content Knowledge

Two Studies with Secondary Teachers

Mathematics Teachers' Exemplification Abilities

- Is crafted over years (Craft Knowledge)
- Is related to PCK, Knowledge of Mathematics, Knowledge of Students' Learning

Instructional Examples

"Any theory of learning which does not deal with how learners and teachers act with, and on, examples is likely to be incomplete as far as mathematics is concerned" (Bills & Watson, 2008, p.77)

Roles and Categorisation of Instructional Examples

Theories Relevant to Exemplification

Empirically-based Guiding Principles

Dangers Underlying Poor Choice of Examples

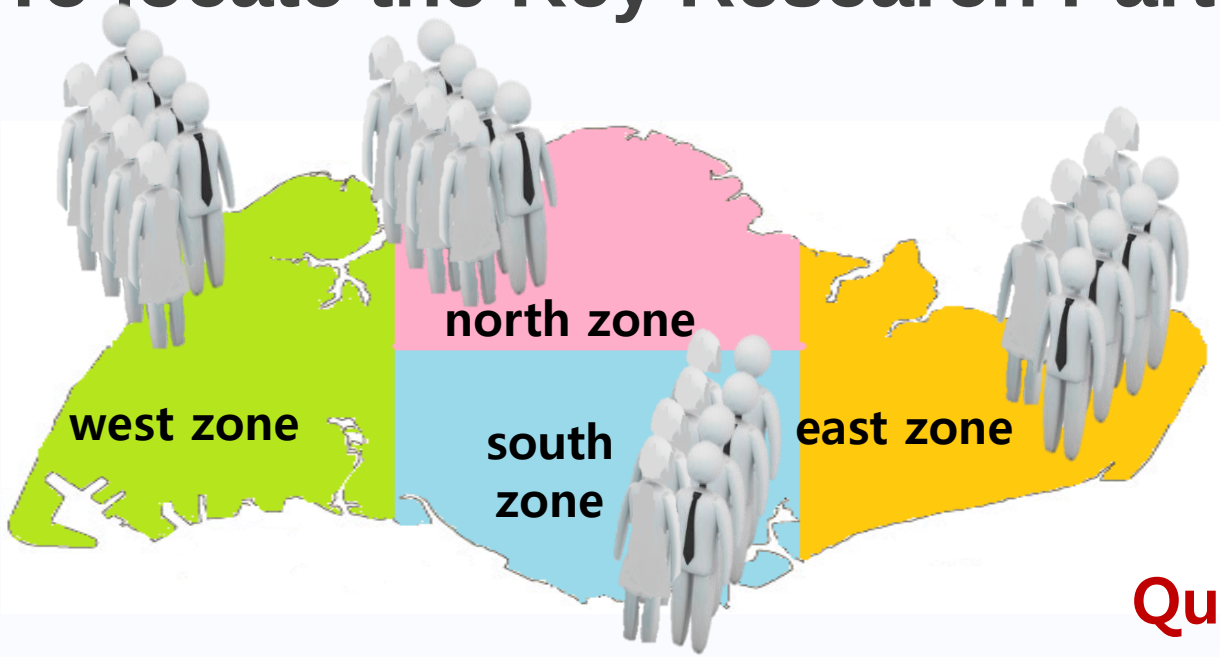
Key Research Participants

Multiple Case Studies

- Current Secondary Mathematics Teachers in **Singapore** (Grade 7-10/11)
- Taught Mathematics at Secondary level in Singapore for at least **five** consecutive years
- Some experience teaching at **Upper** Secondary level (express)

Methodology

To locate the Key Research Participants:



Teacher Questionnaire

From each of the secondary schools, mathematics teachers who

- have taught mathematics for at least **5** consecutive years
- have experience teaching at upper secondary level

In-depth Research

- **Example Use**
- **Mathematics Knowledge for Teaching**
- **Mathematical Beliefs**

Questionnaire Respondents

- Questionnaire
  - 121 Teachers (out of 128 returns)
  - 24 Secondary Schools
- Sample
  - Mean of 12 years of teaching experience
  - 89 had experience in teaching Additional Mathematics
  - 77 taught Mathematics only and the rest taught one other subject
  - All had a first degree and a teaching qualification
  - 25 had a masters degree, 19 were in Mathematics/ Mathematics Education
  - 57 female teachers, 64 males teachers

Age	<30	30-39	40-49	50-59	60+
No. of teachers	7	58	32	17	5

Research Questions

RQ1: How do secondary mathematics teachers choose examples for introducing new mathematical ideas?

RQ2: How do secondary mathematics teachers select homework task(s)?

RQ3: What are the characteristics of a "good" mathematical example in secondary teachers' perceptions?

RQ1. How do Secondary Mathematic teachers choose examples for introducing new mathematical ideas?

List down two factors you consider when selecting examples to **introduce** a new concept/procedure/rule/principle.

From the literature ~

- Skemp (1971) advised educators to reduce the 'noise' in examples during concept formation
- "Start with a simple or familiar case" was the most common category with 193 instances in the 474 identified considerations teachers have when choosing examples (Zodik & Zaslavsky, 2008)
- "Keeping things simple" is a key message that mentors gave during group discussions (Bills & Bills, 2005)

RQ2. How do Secondary Mathematic teachers select homework task(s)?

List down two factors you consider when selecting homework tasks.

From the literature ~

- Hiebert et al. (1996) proposed that teachers look for tasks that
  (1) offer situations that students will perceive as problematic
  (2) provide platforms for students to consider important mathematics
  (3) connect to students' knowledge so that they are attainable

- Mathematical tasks were categoried as (Staub & Stern, 2002, p. 348)
  - Performance-oriented: "tasks requiring only factual knowledge, procedures or algorithms, and outcomes of a procedure".
  - Structure-oriented: "require meaningful knowledge in terms of explicit knowledge of mathematical principles or the application of such principles"

RQ3. What are the characteristics of a "good" mathematical example in Secondary teachers' perceptions?

List down three characteristics of what you think a "good" example would have.

From the literature ~

- Zaslavsky and Lavie (2005) defined a "good instructional example" as one that "conveys to the target audience the essence of what is meant to exemplify or explain" (p. 3). A "good instructional example" is:
  1. Transparent,
  2. Able to foster generalization, and
  3. Should aid in explaining and resolving mathematical subtleties
- The NCTM (2000, p. 52) standards affirmed good problems as those that will
  1. Integrate multiple topics,
  2. Involve significant mathematics,
  3. Help students to solidify and extend their knowledge, and
  4. Stimulate new learning

Category Code	Category Description	Teach Mathematics Idea (%)	Select Homework (%)	Good Example (%)
SA	Students' Abilities	25.5	17.5	13.1
DL	Difficulty Level	21.3	23.1	16.1
FC	Familiar Context	18.3	-	8.36
LO	Learning Objectives	8.09	8.12	5.97
EC	Exemplify Content	8.09	-	10.7
VE	Variety of Examples	6.81	19.2	10.1
CE	Clarity of Examples	5.11	-	15.8
TI	Thinking and Interesting	3.83	-	9.25
CM	Common Misconceptions	2.13	0.855	4.18
CH	Classwork and Homework	0.851	5.98	-
NE	Number of Examples	-	9.83	-
RL	Reinforce Learning	-	8.97	4.78
AU	Assess Understanding	-	6.41	1.49

RQ1. How do Secondary Mathematic teachers choose examples for introducing new mathematical ideas?

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NE	*Number of Examples*	-
RL	*Reinforce Learning*	-
AU	*Assess Understanding*	-

- 235 teachers' considerations were gathered

1. **Student Abilities (SA) (60)**
   - Students' Abilities
     - "pitched to students' current level of understanding"
   - Prior Knowledge
     - "build on concepts they have already learnt"
   - Scaffold Students' Learning
     - "start with numerical examples then algebra to generalise"
2. **Difficulty Level (DL) (50)**
   - Difficulty Level
     "degree of difficulty", "complexity of examples"
   - Simple First Example(s)
     "does not confuse students", "basic question"
3. **Familiar Context (FC) (43)**
   - Students' Personal Experiences
     "relevance to their experiences"
   - Real-life
     "authentic", "linked to real world situations"

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4. **Learning Objectives (LO) (19)**
   - Relevant
     - "examples are linked to concept"
   - Instructional Objectives
     - "required in syllabus", "similar to O/N level questions"
4. **Exemplify Content (EC) (19)**
   - "should convey the essentials of concepts involved"
5. **Variety of Examples (VE) (16)**
   - "standard vs non-standard", "different types of applications"
6. **Clarity of Examples (CE) (12)**
   - "should not involve numbers that are intimidating"
7. **Thinking and Interesting (TI) (9)**
   - "arouse students' interest", "stimulate pupils' thinking"
8. **Common Misconceptions (CM) (5)**
   - "address misconceptions", "questions that show limitations"
9. **Classwork & Homework (CH) (2)**
   - "help them solve questions given for homework later"

RQ1. How do Secondary Mathematic teachers choose examples for introducing new mathematical ideas?

- Top three categories, SA, DL and FC made up 60% of the teachers' considerations
- Key Theme: Build on students' prior knowledge or provide scaffolding (SA) by presenting them with either an easy first example (DL) or a example with familiar context (FC)
- SA: mentors in Bills and Bills' (2005) study also advocate instructional scaffolding via examples and the deliberate sequencing of examples was also observed in teachers in Rowland's (2008) study
- DL: "Easy First Example" was also the most common categories in similar studies: "Keeping things simple" (Bills & Bills, 2005), "Start with a simple/familiar case (FC)" (Zodik & Zaslavsky, 2008)

RQ2. How do Secondary Mathematic teachers select homework task(s)?

Code	Category Description	Select HW (%)
DL	Difficulty Level	23.1
VE	Variety of Examples	19.2
SA	Students' Abilities	17.5
NE	Number of Examples	9.83
RL	Reinforce Learning	8.97
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AU	Assess Understanding	6.41
CH	Classwork and Homework	5.98
CM	Common Misconceptions	0.855
FC	*Familiar Context*	-
EC	*Exemplify Content*	-
CE	*Clarity of Examples*	-
TI	*Thinking and Interesting*	-

- 234 teachers' considerations were gathered

1. **Difficulty Level (DL) (54)**
   - "mixture of easy and difficult level", "1 or 2 higher-order questions to challenge students"
2. **Variety of Examples (VE) (45)**
   - "expose students to different ways of questioning", "involve the same concept in different contexts"
3. **Students' Abilities (SA) (41)**
   - "provide scaffolding"
4. **Number of Examples (NE) (23)**
   - "time required to complete homework", "quantity"
5. **Reinforce Learning (RL) (21)**
   - "reinforcement of concepts", "acquire proficiency"
6. **Learning Objectives (LO) (19)**
   - "SIOs", "relevance of questions"
7. **Assess Understanding (AU) (15)**
   - "check for students' understanding", "tasks should give me feedback on students' learning"

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8. **Classwork & Homework (CH) (14)**
   - "questions similar to teaching examples", "relevance to examples given in class"

9. **Common Misconceptions (CM) (2)**
   - "questions that can surface common mistakes or misconceptions"

- FC, EC, CE, and TI were not considered

RQ2. How do Secondary Mathematic teachers select homework task(s)?

- Top three categories, DL, VE and SA made up 60% of the teachers' considerations
- DL: an assortment of examples of varying difficulty level, the routine ones are meant for practice (Rowland et al., 2003; Watson & Mason, 2004), whereas the demanding tasks are to extend learning (Hiebert et al., 1996).
- VE: Similarly, besides varying the difficulty level, ensuring a diversified array of examples is also essential to increase the ir example space and to provide a fuller understanding. Furthermore, Cai (1997) stressed that "the use of a variety of mathematical tasks can capture a range of students' mathematical performance" (p. 9).
- SA: Hiebert et al. (1996, p. 16) considered SA as vital too as teachers should select tasks that "students can see the relevance of the ideas and skills they already possess".

RQ3. What are the characteristics of a "good" mathematical example in Secondary teachers' perceptions?

Code	Category Description	Good Example (%)
DL	Difficulty Level	16.1
CE	Clarity of Examples	15.8
SA	Students' Abilities	13.1
EC	Exemplify Content	10.7
VE	Variety of Examples	10.1
TI	Thinking and Interesting	9.25
FC	Familiar Context	8.36
LO	Learning Objectives	5.97
RL	Reinforce Learning	4.78
CM	Common Misconceptions	4.18
AU	Assess Understanding	1.49
CH	*Classwork and Homework*	-
NE	*Number of Examples*	-

- 335 teachers' considerations were gathered

1. **Difficulty Level (DL) (54)**
   - Easy to Understand
     "basic", "straightforward", "direct"
   - Extend Learning
     "difficult parts that stretch pupils' mastery", "indirect"

2. **Clarity of Examples (CE) (53)**
   - Clear
     "no ambiguity", "concise"
   - Presentation
     "language used is pegged at students' level"
   - Ease in Calculation
     "not tedious", "does not have many alternative solutions"

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3. **Students' Abilities (SA) (44)**
   - Students' Abilities
     "pitched at the right level for the class"
   - Prior Knowledge
     "able to link with pre-requisites"
   - Scaffold
     "concrete-pictorial-abstract", "from direct to complicated"
   - Diagram
     "visual diagram that assist in conceptualization"
4. **Exemplify Content (EC) (36)**
   - "representative", "clearly explains a concept", "specific"
5. **Variety of Examples (VE) (34)**
   - "wide range of examples", "link to other topics", "applications"
6. **Thinking & Interesting (TI) (31)**
   - "questions that involve inference", "able to 'spark" discussion"

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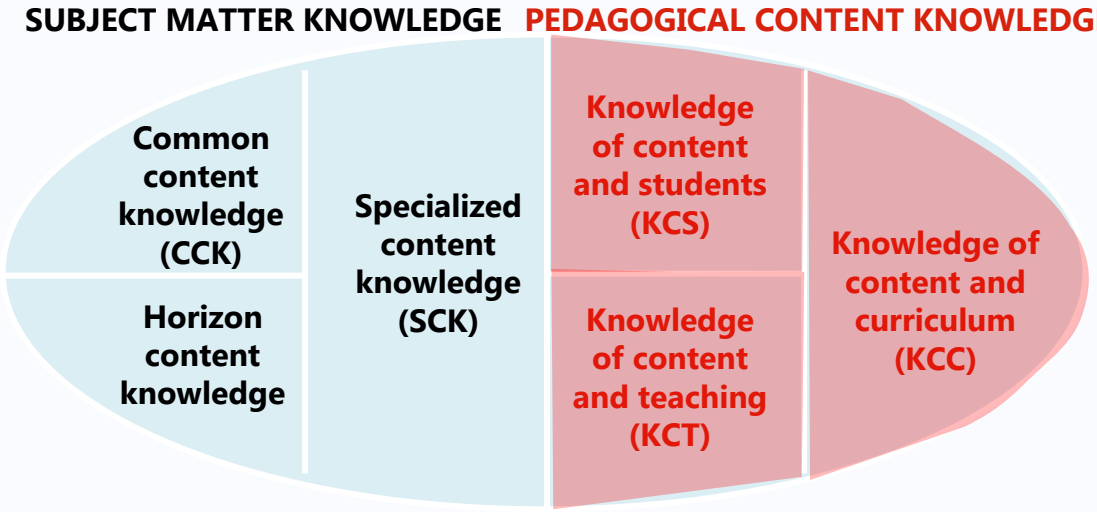
7. **Familiar Context (FC) (28)**
   - "authentic", "applicable to real-life situations"
8. **Learning Objectives (LO) (20)**
   - "exam type of questions", "delivers the lesson objectives"
9. **Reinforce Learning (RL) (16)**
   - "allow students to understand the concept better"
10. **Common Misconceptions (CM) (14)**
    - "many possibilities for common errors to be demonstrated"
11. **Assess Understanding (AU) (5)**
    - "provide good feedback about students' understanding"

RQ3. What are the characteristics of a "good" mathematical example in Secondary teachers' perceptions?

- The bipolar responses in DL are reflective of teachers' considerations at different stage of instruction, easy at first, and thereafter extend learning
- CE is indicative of teachers' intent to reduce the "noise" of a n example (Skemp, 1971) and to allow "one to see the general through the particular" (Mason & Pimm, 1984). This is consistent with the use of "transparent" examples, examples that can "convey to the target audience the essence of what it is meant to exemplify or explain (Zaslavsky & Lavie, 2005).
- DL and SA remained pivotal when teachers select examples (across three RQs)

Aspects of Teacher Knowledge



Knowledge of Content and Students (KCS)

- Consideration for students' prior knowledge (SA)
- Awareness of difficulty level of topics (DL)
- Knowledge of students' conceptions and misconceptions (CM)
- Choice of examples with Familiar Context (FC), Interesting Content (TI), not clouded by other concepts, numerical/algebraic manipulation (CE)

Knowledge of Content and Teaching (KCT)

- Select examples that exemplify the Mathematical idea (EC)
- Sequence homework tasks in "ascending difficulty" to scaffold learning (SA)
- Prefer homework tasks that reinforce classroom teaching (RL/CH)
- Include challenging tasks to deepen understanding (DL)

Knowledge of Curriculum (KCC)

- Prepare students for assessment and address learning objectives (LO)
- Assess students' understanding (AU)

Conclusion

- Limitations
  - Self-response Questionnaire Bias
  - What is written might not be translated into actual lessons
- Guiding Factors in Example Choice
  - Students' Abilities
  - Difficulty Level of Examples
- Example Selection is guided by Instructional Considerations
  - When introducing new content, teachers favoured examples that connect with students' experiences
  - When choosing homework tasks, teachers are more concerned with providing students with varied exposure
- Points to a connection between teacher knowledge and teacher examples


LEARN