Funded Project

Title:	Polynomials in Graph Theory
AcRf Funding:	RI 05/6 DFM
Principal Investigator:	Dr. Dong Fengming
Team Members:	Prof. Koh Khee Meng (NUS) & A/P Tay Eng Guan (NIE)
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Brief description:	

The partition function of a graph is a polynomial defined on a graph in which each edge is assigned a weight. The complex zeros of partition functions play an important role in statistical mechanics, as the real limit points of these zeros determine the possible points of physical phase transitions. The Tutte polynomial is a special case of the partition function and the chromatic and flow polynomials are special cases of Tutte polynomials. A lot of work on these last two polynomials focuses on finding the locations of their zeros. In the past few years, we have done some work on finding the zeros of these polynomials and are on the cutting edge of this area. The project seeks more collaborative opportunities with experts from other universities with a view to achieving significant breakthroughs in this area.

The main objective is to study the locations of zeros of such polynomials to ascertain how they are related to the combinatorial properties of the underlying graph (planarity, chromatic number, maximum degree, connectivity, etc.).

This research direction also attracts researchers from statistical mechanics. From the point of view of statistical physics, these polynomials are nothing other than the partition functions of standard statistical-mechanical models (e.g. the Ising model, the Potts model, the lattice gas, the monomer-dimer model) living on the graph. Although the problems being investigated arise naturally from combinatorics, the key elements of the intuition needed for their solution typically come from statistical physics, in particular from the theory of phase transitions and critical phenomena.

We are particularly interested in some famous unsolved problems in this area and will try to find new tools to attack them. These problems include Welsh's conjecture that there is a constant c such that any flow polynomial has no real zeros in (c, ∞), Sokal's conjecture that any graph has no real chromatic zeros larger than its maximum degree, and Birkhoff and Lewis' conjecture that any loopless planar graph has no chromatic zeros in the interval (4,5). The last one is related to a very famous mathematical conjecture: the four-colour-conjecture, which was first proved in 1976 by the assistance of computers after it had been proposed for more than 100 years.

My publication can be found from the address:
(http://math.nie.edu.sg/fmdong/Research/Publications-Dong.htm)