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A zero-free interval for chromatic polynomials of nearly 3-connected plane graphs*

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Abstract

Let $G = (V, E)$ be a 2-connected plane graph on n vertices with outer face C such that every 2-vertex cut of G contains at least one vertex of C. Let $P_G(q)$ denote the chromatic polynomial of G. We show that $(-1)^n P_G(q) > 0$ for all $1 < q \leq 1.2040....$ This result is a corollary of a more general result that $(-1)^n Z_G(q, \mathbf{w}) > 0$ for all $1 < q \leq 1.2040...$, where $Z_G(q, \mathbf{w})$ is the multivariate Tutte polynomial of G, $\mathbf{w} = \{w_e\}_{e \in E}$, $w_e = -1$ for all e which are not incident to a vertex of C, $w_e \in W_2$ for all $e \in E(C)$, $w_e \in W_1$ for all other edges e, and W_1, W_2 are suitably chosen intervals with $-1 \in W_1 \subset W_2 \subseteq (-2, 0)$.

Keywords: planar graph, Potts-model partition function, multivariate Tutte polynomial, chromatic polynomial, zeros

1 Introduction

The study of chromatic polynomials of graphs was initiated by Birkhoff [3] for planar graphs in 1912 and, for general graphs, by Whitney [14, 15] in 1932. Inspired by the 4-Colour Conjecture, Birkhoff and Lewis [4], obtained results concerning the distribution of the real zeros of chromatic polynomials of planar graphs and made the stronger conjecture that chromatic

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