On atom–bond connectivity index of connected graphs

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Abstract

The atom–bond connectivity (ABC) index of a graph G is defined as

$$ABC(G) = \sum_{uv \in E(G)} \sqrt{\frac{d_u + d_v - 2}{d_u d_v}},$$

where $E(G)$ is the edge set and d_u is the degree of vertex u of G. We give a upper bound for ABC index of connected graphs with fixed number of vertices and maximum degree, and characterize the extremal graphs. We determine the n-vertex unicyclic graphs with the maximum, the second, the third and the fourth maximum ABC indices, and the n-vertex bicyclic graphs with the maximum and the second maximum ABC indices respectively for $n \geq 5$.

Keywords Atom–bond connectivity index; Maximum degree; Unicyclic graphs; Bicyclic graphs; Pendent vertices

1   Introduction

Let G be a simple graph with vertex set $V(G)$ and edge set $E(G)$. For $u \in V(G)$, $\Gamma(u)$ denotes the set of its neighbors in G and the degree of u is $d_u = |\Gamma(u)|$. The atom–bond

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