On the lower bound of the number of perfect matchings of line graphs

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Abstract

In 1970s Lovász and Plummer conjectured that for $k \geq 3$ there exist constants $c_1(k) > 1$ and $c_2(k) > 0$ such that every k-regular elementary graph (i.e., 1-extendable graph) with 2ν vertices contains at least $c_2(k)c_1(k)^\nu$ perfect matchings, where $c_1(k) \to \infty$ if $k \to \infty$. This conjecture was verified for bipartite graphs by Schrijver. In this paper, we show that if G is a connected graph with an even number of edges, then the line graph $L(G)$ of G has at least $2^{|E(G)|-|V(G)|+1}$ perfect matchings, where $V(G)$ and $E(G)$ are the vertex set and edge set of G, respectively. The connected graphs G whose line graphs have exactly $2^{|E(G)|-|V(G)|+1}$ perfect matchings are determined. As applications, we show that the number of perfect matchings of $L(G)$ is odd if G is a tree with an odd number of vertices and even otherwise. We show that the connected $k-$regular line graphs with an even number of edges have exponentially many perfect matchings and we also enumerate weighted perfect matchings of the weighted line graphs of 3-edge colorable graphs. Finally, we enumerate weighted perfect matchings of weighted Kagomé lattices.

Keywords: Perfect matching; Regular graph; Line graph; Cyclomatic number; Kagomé lattices.

1 Introduction

The graphs considered in this paper may have multiple edges but have no loops, if not specified. For a connected graph G, let $V(G)$, $E(G)$ and $\Delta(G)$ be the vertex set, the edge set and the maximum degree of G respectively. $|V(G)|$ and $|E(G)|$ are called the order

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