Technical Report M2010-07
September 2010
Mathematics and Mathematics Education
National Institute of Education
Singapore

On Real Roots of Flow Polynomials*

F.M. Dong†
Mathematics and Mathematics Education
National Institute of Education
Nanyang Technological University, Singapore 637616

Abstract

This article studies the real roots of the flow polynomial $F(G,\lambda)$ of a bridgeless graph G. Let $W(G)$ be the set of vertices in G of degrees larger than 3. For any integer $k \geq 0$, let ξ_k be the largest real number in $(1,2]$ such that $F(G,\lambda)$ has no real zeros in $(1,\xi_k)$ for all graphs G with $|W(G)| \leq k$. We show that ξ_k can be determined by considering a finite set of graphs and therefore deduce that $\xi_k = 2$ for $k \leq 2$, $\xi_3 = 1.430\cdots$ and $\xi_4 = 1.361\cdots$. We also show that for any bridgeless graph G, if $W(G)$ is dominated by some component of $G - W(G)$, then $F(G,\lambda)$ has no roots in $(1,2)$. This result implies that $F(G,\lambda)$ has no zeros in $(1,2)$ whenever $|W(G)| \leq 2$.

Keywords: matroid, graph, characteristic polynomial, chromatic polynomial, flow polynomial, root

1 Introduction

Following Tutte [14] (also see Brylawski and Oxley [2]), the *flow polynomial* of a graph $G = (V,E)$ is the polynomial defined as

$$F(G,\lambda) = \sum_{E' \subseteq E} (-1)^{|E|-|E'|} \lambda^{|E'|+c(E')-|V|}, \tag{1.1}$$

where $c(E')$ is the number of components of the spanning subgraph (V,E') of G. This definition is equivalent to the following basic properties of $F(G,\lambda)$ (see [14] also):

$$F(G,\lambda) = \begin{cases} 1, & \text{if } E = \emptyset; \\ F(G_1,\lambda)F(G_2,\lambda), & \text{if } G = G_1 \cup G_2; \\ 0, & \text{if } G \text{ has a bridge;} \\ (t-1)F(G-e,\lambda), & \text{if } e \text{ is a loop;} \\ F(G/e,\lambda) - F(G-e,\lambda), & \text{otherwise,} \end{cases} \tag{1.2}$$

where G/e and $G - e$ are the graphs obtained from G by contracting e and deleting e respectively, and $G_1 \cup G_2$ is the disjoint union of graphs G_1 and G_2.

*Partially supported by NIE AcRf funding (RI 5/06 DFM) of Singapore.
†Corresponding author. Email: fengming.dong@nie.edu.sg.