

The 3-connectivity of a Graph and the Multiplicity of Zero ‘2’ of its Chromatic Polynomial*

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Abstract

Let G be a graph of order n , maximum degree Δ and minimum degree δ . Let $P(G, \lambda)$ be the chromatic polynomial of G . It is known that the multiplicity of zero ‘0’ of $P(G, \lambda)$ is one if G is connected; and the multiplicity of zero ‘1’ of $P(G, \lambda)$ is one if G is 2-connected. Is the multiplicity of zero ‘2’ of $P(G, \lambda)$ at most one if G is 3-connected? In this paper, we first construct an infinite family of 3-connected graphs G such that the multiplicity of zero ‘2’ of $P(G, \lambda)$ is more than one, and then characterize 3-connected graphs G with $\Delta + \delta \geq n$ such that the multiplicity of zero ‘2’ of $P(G, \lambda)$ is at most one. In particular, we show that for a 3-connected graph G , if $\Delta + \delta \geq n$ and $(\Delta, \delta_3) \neq (n - 3, 3)$, where δ_3 is the third minimum degree of G , then the multiplicity of zero ‘2’ of $P(G, \lambda)$ is at most one.

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